

AN EQUILIBRIUM MODEL OF INDEX FUTURES PRICING

BY

SOUSHAN WU

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN
PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA

1984

ACKNOWLEDGMENTS

Intellectual development is a contract: it gives you leverage, and like an index futures contract, it is marked to the market with some "safty-deposit" margin. The initializer of my development contract is also the chairperson of my committee, Dr. Robert C. Radcliffe, to whom I wish to express my sincere gratitude for his extensive discussion and encouragement. Furthermore, the initial margin of my development contract is reduced by Dr. M.Y. Tarng, Dr. C.C. Chu, Dr. C.Y. Ho, Dr. Tony Lai, Dr. M.P. Narayanan, Dr. Roger Huang and Dr. Raymond Chiang. Their encouragement, valuable comments, and intellectual stimulation made this dissertation possible. Of course, the highest leverage is further implemented by Dr. A.A. Heggsted, Dr. Heim Levy, Dr. Richard Pettway, Dr. Raymond Chiang, and Dr. Rodger Huang, who have instilled in me knowledge of finance both in the class as well as through projects. Special thanks go to Dr. S.R. Cosslett, from whom I have learned more than econometrics, and to Dr. Raymond Chiang, from whom I have learned more than merely finance.

A special acknowledgment of gratitude is due to the National Science Foundation of the Republic of China, the National Chiao-Tung University and the Department of Finance of the University of Florida, which have held a substantial equity interest in this venture. In providing the financial risk capital, these institutions showed overwhelming faith in me.

To my grandfather, Mr. Tu-Chen Wu, I also wish to express my deepest gratitude. His spiritual support and constant encouragement have contributed substantially to my study over the years.

Finally, I would like to thank my wife, Mei-Lin Chang. Without her understanding and encouragement, my study at the University of Florida would not have been possible.

TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS.....	ii
KEY TO SYMBOLS.....	v
ABSTRACT.....	viii
CHAPTER I MOTIVATION OF THE STUDY.....	1
CHAPTER II INTRODUCTION TO STOCK INDEX FUTURES.....	5
Contract Description.....	5
Review of Literature.....	11
Note.....	22
CHAPTER III THE PRICING OF THE INDEX FUTURES.....	23
Notations, Assumptions, and Definitions.....	23
Model.....	28
Comparative Static Analysis.....	34
Summary.....	38
Notes.....	40
CHAPTER IV THE EMPIRICAL STUDY.....	41
A Preliminary Result.....	43
Basic Discussion: Measurement and Data Sets.....	44
Test Methodology and Results	49
Summary.....	64
Notes.....	66
CHAPTER V CONCLUSION.....	83
APPENDICES	
A DATA BASES.....	85
B RESULTS OF FIRST STAGE LEAST SQUARE.....	88
REFERENCES.....	94
BIOGRAPHICAL SKETCH.....	99

KEY TO SYMBOLS

t	- Current period
T	- Maturity of the index futures contract
$F(t,T)$	- Current futures price maturity at T .
$\tilde{F}(T,T)$	- Maturity futures price
$S(t)$	- Value of the index-equivalent portfolio at t (which is formed as the same value as of the spot value $I(t)$)
$\tilde{I}(T)$	- Value of the spot index at time T
$\tilde{D}$	- Total cash received from dividend payments on the index-equivalent portfolio during the period t to T
$\tilde{S}(T)$	- Value of the index-equivalent portfolio at T which is formed at t , includes dividend, equal to $I(T) + D$
C_{pl}	- Cost of being long the index-equivalent portfolio
C_{ps}	- Cost of being short the index-equivalent (which may include foregone interest on the short sales proceeds)
C_{fs}	- Cost of being long the index futures
C_{fl}	- Cost of being short the index futures
X_{si}	- Number of endowment units which investor i holds of the index equivalent portfolio (This is exogenously given)
X_s	- Total number of endowment units of the index-equivalent portfolio in the economy
X_{si}	- Number of units of the index portfolio held by the investor i
X_{fi}	- Net long number of units of index futures contract held by the investor i
C_i	- Initial cash of investor i
b_i	- Number of units of bond held by the investor i
r_f	- Risk-free rate during this period (t,T)
η_i	- Risk tolerance of investor i
η	- Risk tolerance in the economy
W_{ti}	- Wealth of the investor i at time t which can be put among bonds, index-equivalent portfolio and the index futures

$\tilde{W}_{Ti}$	- Terminal wealth of investor i at time T
E_i	- Expected value of the terminal wealth of investor i at time T
σ_i^2	- Variance of terminal wealth of investor i at time T
$G_i(E_i, \sigma_i^2)$	- Preference function of investor i, which is determined by the expected value and the variance of the terminal wealth
σ_I^2	- Variance of the spot index at time T
σ_D^2	- Variance of the dividend payout which is carried over in the spot index-equivalent portfolio
σ_{ID}	- Covariance of the spot index and the random dividend payout over period
σ_s^2	- Variance of the index-equivalent portfolio price
σ_F^2	- Variance of the index futures price
σ_{sF}	- Covariance of the index futures price and index-equivalent portfolio price
σ_{FD}	- Covariance of the index futures price and random dividend payout
$\tilde{r}_F$	- Percentage price change of the index futures
$\tilde{r}_S$	- Rate of return on the index-equivalent portfolio
$\tilde{R}_F$	- Percentage price change of the index futures in terms of spot index
ρ	- Correlation coefficient of the index futures price and the index-equivalent portfolio
S&P 500	- Standard and Poor's Composite Average Index
VLA	- Value Line Composite Average Index
NYSE	- New York Stock Exchange
NYFE	- New York Futures Exchange
AMEX	- American Stock Exchange
OTC	- Over the Counter
CRSP	- Center of Research in Security Prices, University of Chicago
COMPUSTAT	- Standard & Poor's Compustat Service, Inc.

FD - Futures Discount

FCESP - Future Certainty-Equivalent Spot Price

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree of Doctor of Philosophy

AN EQUILIBRIUM MODEL OF INDEX FUTURES PRICING

By

Soushan Wu

December 1984

Chairman: Robert C. Radcliffe

Major Department: Finance, Insurance and Real Estate

The objectives of this study, all pertaining to pricing efficiency, are as follows: (1) to develop a theory of cash-futures price relationships for stock index futures which includes dividend risk within the framework of the partial equilibrium approach; (2) to test whether a dividend risk premium has affected cash-futures price relationships; and (3) to provide a framework for evaluating the efficiency and hedging in the pricing of stock index futures. Two types of evidence are employed to attain these objectives. First, the principles of cash-futures price behavior derived from observation of long-established futures markets serve as points of departure, with substantial consideration given to certain fundamental differences that arise from the specification of stock index futures. Second, empirical tests of the hypothesized equilibrium pricing are evaluated.

This study demonstrates that when the dividend risk is taken into consideration, a covariance term between the dividend payout and the index-equivalent portfolio is added to the Perfect Market Model developed by an arbitrage approach. The study also shows that the dividend risk premium

empirically increases with seasonality when time to the maturity increases. More-over, in the last thirty trading days, the market bears a dividend risk level which is statistically insignificant.

CHAPTER I MOTIVATION OF THE STUDY

With the existence of exchange markets, individuals are able to hold their personal endowment in terms of current consumption and investment for the future to consume. The exchange can be improved by enlargening the set of feasible patterns of consumption and investment over time and by sharing of uncertainty associated with consumption and investment in the future. With such a market, satisfaction is increased for the individual and for the society as well.

A new financial instrument, the stock index futures contract, has recently been introduced to facilitate intertemporal allocation of resource and risk sharing (since February 1982). This index futures market, like other existing futures markets, claims to provide two major advantages to market participants: (1) risk transfer and (2) price discovery (Garbade & Silber, 1983). Both effects allow the capital market to become more complete (Ross, 1976). Risk transfer refers to a hedger's ability to transfer price risk to another hedger or to a speculator (if no offsetting hedger can be found). Specifically, the index futures market can separate market risk from unsystematic risk. Price discovery refers to the information available in explicit prices of the trading transaction. Both the pricing discovery and risk transfer have the ability to extend the range of investment and risk management strategies.

These two functions also result in conceptual economic models which relate existing futures prices to existing spot prices.

Conceptual models together with empirical tests which relate spot price and futures are very common in commodity contracts (although results of such models and tests are debatable). However, within the past few years in which trading has begun on stock index futures contracts, little theoretical modeling or empirical testing has been conducted. The studies of stock index futures to date suggest a puzzle: the actual stock futures prices do not conform with existing theoretical pricing models which are based upon a perfect market pricing model under the assumption of a nonstochastic dividend. The perfect market pricing model implies that the index futures price should be equal to the future certainty equivalent of today's spot price (at the maturity date) minus the accumulated dividend payout over this period (Cornell & French, 1983).

This dissertation is motivated by the fact that existing theoretical models of stock index futures do not stand up to empirical facts. In the earliest period of their trading (February 1982 to September 1983), actual stock index futures consistently sold for less than their "theoretically" derived values based on perfect market model (Figlewski, 1983a). A number of possible explanations have been offered by Modest and Sundaresan (1983), Cornell and French (1983), and Figlewski (1983a). Among these studies, Figlewski (1983a) classified the potential reasons as the so-called (market) equilibrium argument and disequilibrium argument. But research on these new financial instruments has just begun and none of the existing studies has been able to adequately explain the differences between actual stock index futures price and "theoretical" price of perfect market model.

Researchers reach their own conclusions by examining some specific dates. In the most recent study, Figlewski (1984) employs more complete data to argue that this futures market is in a long-run disequilibrium in

its early stage. Both Modest and Sundaresan's "short sales not fully used" argument (1983) and Cornell and French's "tax-related timing option" (1983) employ a cash-futures arbitrage approach to obtain their pricing relationships of index futures based on the market equilibrium approach. And in both of these equilibrium models, there is an assumption that dividends to be paid on the spot index-equivalent portfolio are known with certainty.

However, if the dividend to be paid on the spot index is unknown in advance, then the strategy of "long index-portfolio and short index futures one unit each" is not sufficient to completely hedge the risk associated with such an unknown dividend payment. If an allowance is made for an unknown dividend payment, it is theoretically plausible that the index futures contract should be priced lower than what the so-called "perfect market" model suggests. This discount occurs because the individual who is long an index futures does not receive dividend payments, but does face a dividend risk. If the dividend payout is stochastic, index futures and the index-equivalent portfolio may not be of the same systematic risk level. Consequently, the arbitrage approach to pricing breaks down. A perfect arbitrage is not possible and index futures pricing models should not be developed around such arbitrage relationships. Instead, a more general equilibrium approach should be employed.

The objectives of this study are two: (1) to derive and (2) to test a closed form equilibrium price for a stock index futures contract, which includes the effect of unknown dividends. This closed-form equilibrium futures price will also be linked with traditional Capital Assets Pricing Model (CAPM) framework and conventional hedging strategies after some rearrangement.

Following a basic review of index futures and a synthesis of past

studies in Chapter II, a basic pricing model with comparative statics analysis and interpretation is presented in Chapter III. Chapter IV provides econometric justification for the form of this predictive model and fits the forecast to actual New York Stock Exchange (NYSE) and Standard and Poor's Composite (S&P 500) index futures based on a sample taken from New York Stock Futures Exchange and The Chicago Board, starting from the first trading contract to the December 1984 contract. Conclusions of this study are presented in Chapter V.

CHAPTER II INTRODUCTION TO STOCK INDEX FUTURES

In this chapter, the legal characteristics of index futures contract are described in the first section. Following this, the various studies relating to this study are briefly reviewed.

Contract Description

A stock index futures contract is an obligation to buy or sell a hypothetical portfolio of all the stocks in an index at a stated price at a certain date. It can be liquidated before maturity.

At present, there are three main actively traded stock index futures contracts: the Value Line Index traded on the Kansas City Board of Trade, the Standard & Poor's 500 traded on the Index and Option market of the Chicago Mercantile Exchange, and the New York Stock Exchange Composite Index traded on the New York Stock Exchange. In addition, a New York Stock Exchange Financial Index Futures and other contracts, which are on narrower measures of equity market activity such as Utility and Transportation indices, have recently been introduced. Options on these three main indices are also available now.

An index futures contract is similar to other kinds of commodity futures contracts paid for on a unique installment plan. The investor who buys a futures contract agrees to buy one unit of a financial product at a specified maturity time at a specified futures price. The index future price is determined when the contract is written and

is specified in the contract. The stock index futures price should differ from current or expected future (t , or T) stock index prices for at least two reasons. First, the futures price is chosen so that no payment is made when the contract is written; i.e., at the initiation phase the futures contract has zero market value. But as the contract matures, the investor must make or receive daily installment payments toward the eventual purchase of the financial product. This is referred to as "marking-to-market." The total of the daily installments and the payment at maturity will equal the futures price specified when the contract was initiated. Second, the futures trader does not receive the dividends that are paid to the stockholder, but faces a dividend risk.

What makes futures contracts unique among installment plans is that the daily installments are not specified in advance in the contract, but are determined by the daily change in the futures price. If the futures price rises, then the investor, who is long the futures contract, receives a payment from the investor, who is short. The payment is the rise in the futures from the previous day. On the other hand, if the futures price falls, the long holder pays the short holder the change in the daily futures price.

The effect of marking-to-market is to rewrite the futures contract each day at the new futures price. Hence the value of the futures contract after the daily settlement will always be zero since the value of a newly written futures contract is zero. When the contract matures, the long investor will have already paid or received the difference between the initial futures price and the futures price at maturity time. With these payments to his credit, he will have a balance due equal to the futures price at maturity. But the value of a futures contract

Therefore, at maturity the futures price must equal the current spot price. Thus the balance due is simply the current spot price at the maturity time. Unlike a forward contract, the value of a futures contract-- after settlement--is always zero.

Some special characteristics of index futures contracts are discussed in what follows. The margin on a futures contract represents a "good faith" deposit on the part of the buyer and seller. Margin requirements on stock index futures approximate 5-10% of the contract's value. This results in a high degree of leverage for the futures trader. Futures margins differ significantly from spot stock margins. They are much lower than stock margins and involve no extension of credit or expense on any unpaid (borrowed) balance. In addition, the futures margin must be restored daily during adverse price movements (marked to the market). Traders in a profitable position may withdraw any excess margin. In certain cases, margin can be posted in Treasury Bills, so interest can be earned while the trading program is in place. The delivery process in the settlement of futures contracts serves a positive economic function in that it assures a certain level of convergence between the cash and futures prices at the expiration date. This price correction is essential to the development of a successful futures contract. Cash/futures convergence in the stock index futures is virtually assured by the "cash settlement" procedure in these contracts. Rather than require the delivery of actual stock, all open positions are "marked to the market" at expiration. In addition, the delivery price is not determined in the futures pit, but by the actual cash market close on the last day of trading (like S&P 500 index futures), or the

average of the closes of the last and next last day (like the NYSE index futures case). This assures convergence of the futures price with the price for the underlying commodity, the spot index, and prevents any attempts to "squeeze," "corner," or otherwise manipulate the market. On the last trading day of the expiring contract, all three indices futures are traded until 4:00 P.M. (EST) only.

Do prices in one market have an impact on prices in the other market? With cash settlement, the arbitrage possibility between the cash market and futures market is encouraged. An arbitrager can profit by going long a unit of index futures, short a unit of the index-equivalent portfolio and investing the balance in risk free bonds simultaneously, if the observed index futures price is lower than the "theoretical" price in mind. Meanwhile, he also can profit by shorting a unit of index futures, borrowing at the risk-free rate, and going long a unit of index-equivalent portfolio simultaneously, if the observed index futures price is higher than the "theoretical" price in mind. This cash-futures arbitrage is often referred to as basis speculation and is best represented as a type of "risk arbitrage." It contributes to the economic activity by both the risk transfer and price discovery functions of futures markets.

For taxable investors, under the current tax code, the gain or loss on any futures transaction entered into after June 23, 1981, is be treated as 60 percent long-term capital gains, and 40 percent as short-term, without regard to the period of time for which the position was held. It means that the tax rate of index futures trade is 32% currently, i.e., $(60\% \times 20) + (40\% \times 50) = 32\%$.

Value Line Composite Average (VLA)

Published and maintained by Arnold Bernhard and Co., the VLA is an equally weighted geometric average of about 1700 stock prices expressed in index form (June 30, 1961 = 100.0) Nearly 90 percent of the stocks included in the VLA are traded on the New York Stock Exchange. Based on this spot index, a Value Line Index Futures Contract is specified in Table 2.1.

TABLE 2.1

Value Line Composite Index Futures

Exchange:	Kansas City Board of Trade
Trading Months:	March, June, September, December; 18 months forward
Trading Hours:	10:00 A.M. - 4:15 P.M., N.Y. time
Contract Size:	Futures price x \$500
Minimum Price	
Fluctuations:	5 points, equivalent to \$25
Daily Price Limits:	500 points (\$2,500)
Last Trading Day:	Last business day of contract month
Trading began:	Feb. 24, 1982

Standard & Poor's 500

This index, commonly called the S&P 500, is a broadly based arithmetic average, utilizing the share prices of 500 different companies: 400 industrials, 40 utilities, 20 transportation and 40 financial companies. The market value of the index is approximately 80% market value of all stocks on the New York Stock Exchange. Each stock in the index is weighted to reflect the stock's total influence on the index relative to its market value. To determine the weighting, the number of shares outstanding is multiplied by the price per share. Thus, a stock's total market value determines its importance in this index.

The S&P 500 has been widely accepted as a benchmark for portfolio manager performance as well as a measure of economic activity. It is one of the components of the Index of Leading Economic Indicators. The S&P 500 is based on the average weekly closing value of 1941-1943 and indexed to a value of 10. The stock index futures contract on this index is specified in Table 2.2.

TABLE 2.2

Standardized & Poor's 500 Index Futures

Exchange:	Index and Options Market
Trading Months:	March, June September, December, 18 months forward
Trading Hours:	10:00 A.M. - 4:15 P.M., N.Y. time
Contract Size:	Futures price x \$500
Minimum Price	
Fluctuation:	5 points, equivalent to \$25
Daily Price Limits:	500 points (\$2500)
Last Trading Day:	Third Thursday of contract month
Trading began:	April 21, 1982

New York Stock Exchange Composite

This index is an arithmetic average consisting of all common stocks listed on the NYSE. As with the S&P 500, each stock in the NYSE index is weighted in proportion to the stock's market value. The index is based on the close price of December 31, 1965, and indexed to a value of 50. The composite futures and financial futures based on this are specified as Table 2.3 and Table 2.4.

TABLE 2.3

New York Stock Exchange Index

Exchange:	New York Futures Exchange
Trading Hours:	10:00 A.M. - 4:15 P.M., N.Y. Time
Contract Size:	Futures x \$500
Minimum Price	
Fluctuation:	5 points, equivalent to \$25
Daily Price Limits:	none
Last Trading Day:	Second last business day of contract month
Trading began:	May 6, 1982

TABLE 2.4

New York Stock Exchange Financial Index

Exchange:	New York Futures Exchange
Trading Months:	March, June, September, December; 18 months forward
Trading Hours:	10:00 A.M. - 4:15 P.M., N.Y. time
Contract Size:	Futures price x \$1000
Minimum Price	
Fluctuation:	1 point, equivalent to \$5
Daily Price Limits:	none
Last Trading Day:	Second last business day of contract month. However, effective with December 1984 contract and all subsequent contracts, the last business day will be the third friday of the month.

Review of Literature

To date, three basic arguments have been proposed to explain why actual futures prices are lower than what the perfect market pricing model suggests. These include 1) a tax-related timing option, 2) no full use of the short sales, and 3) long-run disequilibrium in the index futures market in the early stage.

In sum, the tax-related timing option suggested by Cornell and French (1983), and the no full use of short sales suggested by Modest

and Sundaresan (1983) have been classified as equilibrium arguments by Figlewski (1983a). Furthermore, he believes that the market is in disequilibrium stage in the early stage.

In a market of no transaction cost, no tax, dividend certainty, and no information access cost, a perfect market pricing model is developed as follows:

$$F(t,T)=I(t) \exp((r_f-d)) \dots\dots\dots(2.1)$$

where

- F(t,T) : the index futures price at time t, maturity at T.
- I(t) : the index-equivalent portfolio at time t.
- r_f : risk-free rate
- d : dividend rate (total dividend payout from t to T divided by the current spot index).

Due to possible imperfections of the capital market, as Modest and Sundaresan suggest, empirically observed futures discount may be explained by the fact that short sellers of spot seldom obtain full use of the proceeds of the short sales. They argue that if the proceeds of the short sale of spot are not available to be invested in the money market, the "short spot--long futures" arbitrage becomes profitable only when the futures price is below the current spot index by an amount greater than the dividend yield on the index portfolio because a short seller must pay the dividends on the shares he has borrowed. An upper bound and a lower bound thus are developed for index futures pricing. For illustrative purpose Modest and Sundaresan present the bounds and observed futures prices for the June 1982 and December 1982 S&P 500 stock index futures contract. The bounds are presented under the

alternative assumptions that short sellers have zero, half, and full use of the proceeds, both in the no dividend adjustment case and in the dividend adjustment case under the assumption of nonstochastic dividend. Thus, for each contract, six sets of bounds and prices are presented. For the cases where dividends are adjusted, the bounds are given by the eq(2.2) which can be written as

$$\frac{I(t) + C_{pl} + C_{fs} - D}{\exp(-r_f)} \geq F(t,T) \geq \frac{I(t) - C_{ps} - C_{fl} - D}{\exp(-r_f)} \quad \text{.....(2.2)}$$

where

- C_{pl} : Cost of being long in the index-equivalent portfolio
- C_{ps} : Cost of being short in the spot (which may include foregone interest on the short sales proceeds)
- C_{fs} : Cost of being short in the index futures
- $I(t)$: Price of one unit of index-equivalent portfolio at time t
- D : Known dividend with reinvested in risk-free rate
- r_f : Risk-free rate
- $F(t,T)$: Index futures price at t , maturity at T

Based on this equation, the figures for December 82 S&P 500 contract are reproduced here as Figure (2.1 - 2.3) for reference. This argument, according to Figlewski (1983a), only explains why arbitrage might not force the futures discount to disappear once it develops. It gives no insight into why the discount should exist in the first place. This means that the motivation to short the index equivalent portfolio is not clear. Besides, trading index futures involves no investment except some margin requirement which can be met by using securities such

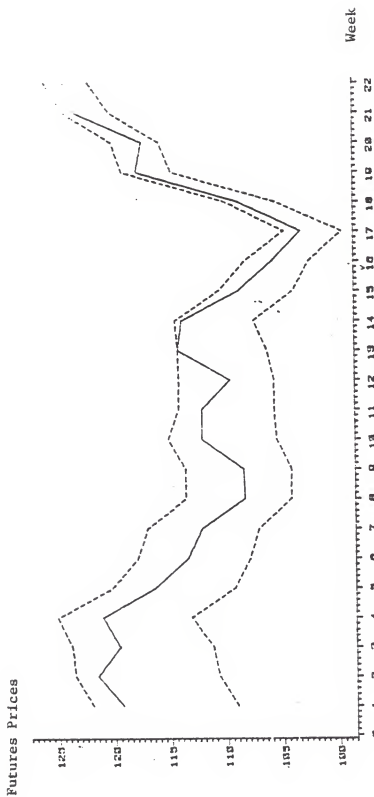


Figure 2.1 S&P 500 Contract Maturities December 1982. Futures prices and bounds (zero use of proceeds, adjusting for dividends).

Source: Modest and Sundaresan (1983) p.30, reproduced by permission.

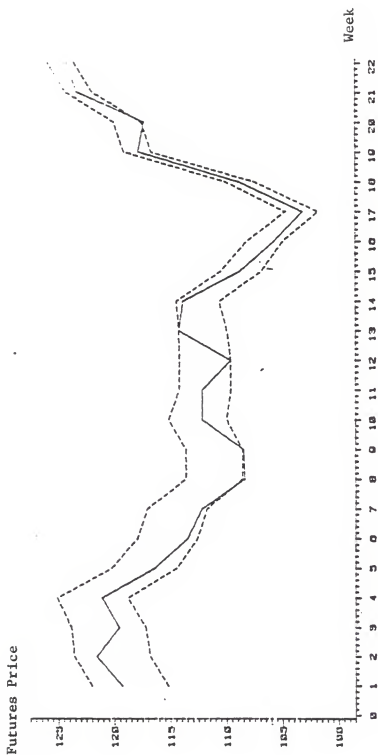


Figure 2.2 S&P 500 Contract Maturating December 1982. Futures prices and bounds (half-use of proceeds, adjusting for dividends). Data plotted for April 21 - September 15, 1982.

Source: Modest and Sundaresan (1983) p.34, reproduced by permission.

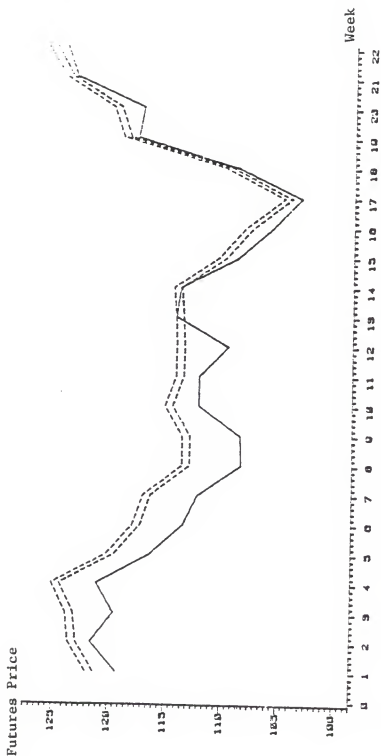


Figure 2.3 S&P 500 Contract Maturities December 1982. Futures price and bounds (full use of proceeds, adjusting for dividends). Data plotted for April 21 - September 15, 1982.

Source: Modest and Sundaresan (1983) p.38, reproduced by permission.

as Treasury Bill which would be held regard less of the futures today. Both "long the spot--short the futures" and "short the spot--long the futures" strategies are different in risk level and investment level.

Cornell and French (1983) point to a difference in the way stock and futures returns are taxed. For index futures all paper profits are taxed as if they had been realized by the end of the fiscal year or by the maturity of the index futures, whichever comes first. Returns on a stock portfolio, however, are taxed as short term gains if realized within one year. But if the holding period is extended beyond one year, the tax rate drops to 40% of the short term rate. Therefore, a stock portfolio offers a "tax-related timing option" that the futures contract does not possess.

According to Cornell and French (1983), the timing option is a right to defer to pay the capital gain taxes. Consider an investor who buys an index-equivalent stock portfolio. If its value goes down in a year, he can sell and deduct the loss at short term rates. On the other hand, if it goes up he has the option to extend his holdings period to take advantage of the long term gains rates. Thus, for a taxable investor who knows the dividend payout in advance, Cornell and French derive a functional form for stock index futures to be priced below their theoretical level by an amount equal to the value of the timing option.¹ Consequently, the relative timing option is defined as the difference between the "theoretical" price and the observed price relative to the "theoretical" price. The relative timing option of some selected

contract is adapted in Table 2.5 for reference. However, the timing option is difficult, if not impossible, to compute a theoretical value (Wu, 1983), if one can not specify how long investors are going to hold the index-equivalent portfolio. In addition, the timing option must be positive. There exist, moreover, a huge volume of stocks held by tax exempt investors and by taxable investors whose holding periods are already greater than one year. If the effect of the timing option on futures prices were the main cause of the futures discount, these investors should be writing timing options by selling their stock and buying index futures and risk-free securities to increase their returns. Furthermore, if the dividend to be received on the index is unknown in advance, the perfect arbitrage argument used to derive the model breaks down.

Besides these two articles, other researchers which make more comments but offer fewer convincing explanations on the index futures discount can be found in Figlewski (1983a). They are all branches of the market "equilibrium" argument.

Figlewski (1983a) presents a permanent market dis-equilibrium argument. He believes that there exists no temporary short run dis-equilibrium in the sense that prices do not adjust to equate supply and demand but that there does exist a long run dis-equilibrium in the index futures market. In other words, actions of investors already in the market create profitable investment opportunities for outside investors who, for one reason or another, are slow to take them up. Reasons for this include an unfamiliarity with the new markets, inertia in developing systems to take advantage of the opportunities they present, legal aspects

TABLE 2.5
Relative Value of the Timing Option^a

Contract Maturity	S&P 500		NYSE	
	Days to Maturity	Relative Value of Option	Days to Maturity	Relative Value of Option
		1 June 1982		
June	18	1.69%	28	1.95%
September	108	2.77%	120	2.97%
December	199	3.52%	211	3.20%
March 83	290	4.00%	304	3.73%
		1 July 1982		
September	77	0.01%	90	0.13%
December	168	0.95%	181	0.83%
March 83	259	2.03%	272	2.06%
June	350	2.93%	363	2.97%
		2 August 1982		
September	45	-0.82%	58	-0.91%
December	136	-0.54%	149	-0.62%
March 83	227	0.14%	240	-0.19%
June	318	0.72%	331	0.36%
		1 September 1982		
September	15	1.12%	28	1.07%
December	106	1.41%	119	1.39%
March 83	197	1.76%	210	1.85%
June	288	2.33%	301	2.42%

Source: Cornell and French (1983) p.691, reproduced by permission.

^aThe relative value of the timing option is expressed as a percentage of the stock price. It is estimated as

$$c(t) = \frac{1 - F(t, T)}{I(t) [e^{(1-i)r(t, T)(T-t) - g} - \int_t^T (1-i)d(w) e^{(1-i)R(t, w, T)(T-w)} dw]} / (1-g)$$

where g: capital gain tax rate
 i: ordinary income rate
 d(w): dividend rate at w
 r: bond forward rate
 R: loan forward rate

of trading futures for cautious portfolio managers, the time required to intergrate accounting procedures for futures trading into daily operations, etc. He regresses the discounts on a time trend and the number of days to expiration, to adjust for the fact that the discount must go to zero as a futures approaches maturity. He observes that futures discounts have been decreasing over time, after taking account of the effect of the time to expiration and the high degree of serial correlation in the relationship. Finally, he believes that the futures market is slowly coming into equilibrium.

In a more recent article, Figlewski (1984) presents some further results in the hedging performance and basis risk on the stock index futures. This empirical study stands on the ground of a "perfect market" pricing model, which is relevant to our topic. However, he argues that the effects of dividend risk, the length of the holding period, and the time remaining to expiration of the futures contract should be considered. He defines the "return" of index futures in terms of the spot index and takes the variance operator to get the hedge ratio which minimizes risk. Furthermore, he states that the risk minimizing hedge ratio is the portfolio's beta coefficient with respect to the market index if the hedge portfolio is held to the maturity and dividends are not random. He reports the effectiveness of Standard and Poor's 500 index futures in hedging major stock index portfolios over a one week holding period in the sample, July 1, 1982, to September 30, 1983. For a one week holding period, hedging a diversified portfolio weighted toward large capitalization stocks can yield fairly good risk reduction, from about 20 to 30 percent of unhedged portfolio's standard deviation. However, hedging

effectiveness is substantially reduced by the presence of unsystematic risk, even in the amount contained in a broadly diversified portfolio of small stocks like the American Stock Exchange (AMEX) and Over the Counter (OTC) portfolios. He concludes that a short duration hedge for an individual stock or a small portfolio might be quite unsatisfactory. Finally, because of basis risk, the minimum risk hedge ratio was less than the portfolio's beta in every case, with the adverse effects of overhedging being more serious for returns than for risk levels.

To explain the basis, Figlewski uses the perfect market pricing model for the futures price to prevent portfolio arbitrage. Price difference between this "equilibrium" price and the actual price is analyzed. The sample is then split into thirds to show the difference in the market's behavior over time. He shows that underpricing of futures was significant in the first third of the sample, but this was not true of the period as a whole for the nearest contracts.

In considering the sources of basis risk in a hedge of the S&P 500 portfolio itself, Figlewski believes that dividend risk was not an important factor, while hedge duration and time to expiration of the futures contract were, to some extent. With regard to the pricing of stock index futures, he finds that the significant underpricing that was widely remarked in the very early months of trading seems to have disappeared, and that deviations from the theoretical pricing relation have diminished. This implies that underpricing does not reflect an equilibrium differential, which a factor like the value of the tax timing option would cause.

In this dissertation, a dividend risk is considered to be a potential factor to explain the futures discount of perfect market model. Because the dividend might be uncertain before it is announced, a dividend risk premium might be priced in the market. Suppose the investor holds a hedge portfolio which consists of index futures and index-equivalent portfolio one unit each, the dividend risk should be considered. If the dividend is indeed unknown, a dividend risk premium explicit in an equilibrium model should be developed.

The following chapter, Chapter III, presents the theoretical setting in which the effect of an unknown dividend is taken into account so as to determine (1) the price relationship between the spot and the index futures, (2) the systematic risk of index futures contract, and (3) the hedging relationship between the spot and the index futures. All three equations are testable and estimatable.

Note

- [1] The value of the timing options increases monotonically with the variance of return, while return variance generally rises following a stock split. Consequently, Cornell (1984) uses stock splits data to test the theory of the timing option in the index futures market. He finds that the timing option is not empirically important or that the expected increase in variance is the same for all stocks.

CHAPTER III THE PRICING OF THE INDEX FUTURES

In the perfect market pricing model, it is assumed that the dividend for the index-equivalent portfolio underlying the index futures is known in advance. Thus, there is capital gain risk but no dividend risk. The implication of this is that the theoretical equation might systematically overprice the index futures. In this chapter, however, index futures pricing is extended to incorporate stochastic dividend yields under a partial equilibrium approach, assuming that market participants are risk averse. Adopting a set of common assumptions, a closed form solution is found, which can link the pricing relationship with the CAPM and with the hedging strategy as well. We discover that a dividend risk premium is required in evaluating the index futures pricing.

To derive these results, a set of notations, definitions, and assumptions are needed. These are presented in the first section of this chapter. In the second section we develop the model necessary for the analysis of futures contracts in the comparative statics analysis of these results. Consequently, several interpretations of this comparative statics analysis are provided in the third section. A summary of this chapter is provided in the forth section.

Notation, Assumptions, and Definitions

The notation, definitions and assumptions used here is similar to that used to derive the CAPM model as in Sharpe (1964), Lintner (1965), and Mossin (1966). We only add several assumptions to facilitate the

inclusion of an index futures contract, such as cash settlement, and stochastic dividend effect. We will also assume that the investor determines his current consumption level first and then emphasizes on the composition and size of his investment portfolio in these three assets: index futures contract, a spot index-equivalent portfolio and a risk free bond. We assume the investor preference function is of the form, $G_i(E_i, \sigma_i^2)$, where E_i and σ_i^2 represent the single-period expected value and variance of value of the i th investor's portfolio of risky assets. We further assume that

$$\frac{\partial G_i}{\partial E_i} > 0, \text{ and } \frac{\partial G_i}{\partial \sigma_i^2} < 0,$$

that is, greater expected values are preferred and variance of value is not.

Notations

t	Current period
T	Maturity of the index futures contract
$F(t, T)$	Current futures price maturity at T
$\tilde{F}(T, T)$	Maturity futures price
$S(t)$	Value of the index-equivalent portfolio at t (which is formed as the same as value of $I(t)$)
$\tilde{I}(T)$	Value of the spot index at time T
$\tilde{D}$	Total cash received from dividend payment on the index-equivalent portfolio during (t, T) period
$\tilde{S}(T)$	Value of the index-equivalent portfolio at T which is formed at t , include dividend, equal to $I(T) + D$
X'_{si}	Number of endowment units which investor i holds of the index equivalent portfolio (This is exogenously given)

X_{si}	Number of units of the index portfolio held by the investor i
X_{fi}	Net long number of units of index futures contract held by the investor i
C_i	Initial cash of investor i
b_i	Number of units of bond held by the investor i
r_f	Risk-free rate during this period (t, T) Risk tolerance of the investor i
W_{ti}	Wealth of the investor i at time t which can be put among bonds, index-equivalent portfolio and the index futures
$\tilde{W}_{Ti}$	Terminal wealth of investor i at time T
E_i	Expected value of the terminal wealth of investor i at time T
σ_i^2	Variance of terminal wealth of investor i at time T
$G_i(E_i, \sigma_i^2)$	Preference functions of investor i , which is determined by the expected value and the variance of the terminal wealth
σ_I^2	Variance of the spot index at time T
σ_D^2	Variance of the dividend payout which is carried over in the spot index-equivalent portfolio
σ_{ID}	Covariance of the spot index and the random dividend payout over period
σ_S^2	Variance of the index-equivalent portfolio price
σ_F^2	Variance of the index futures price
σ_{SF}	Covariance of the index futures price and index-equivalent portfolio price
σ_{DF}	Covariance of the index futures price and random dividend payout
$\tilde{r}_F$	Percentage price change of the index futures
$\tilde{r}_S$	Rate of return on the index-equivalent portfolio
$\tilde{R}_F$	Percentage price change of the index futures in terms of spot index
ρ	Correlation coefficient of the index futures price and the index-equivalent portfolio

Note: "Tilde" is used to denote random variable notation.

And "bar" is used to denote the mean value of random variable.

Assumptions

A1: The investor i deals with only three assets (i.e. index equivalent portfolio, index futures and bonds). The market of the specification is as follows:

<u>Market</u>	<u>Price at t</u>	<u>Future Value at T</u>	<u># held</u>	<u>Aggregate</u>
Spot Market	$S(t)$	$\tilde{S}(T)$	X_{si}	X_s
Index Futures	0	$\tilde{F}(T,T) - F(t,T)$	X_{fi}	0
Bond	$\exp(-r_f)$	1	b_i	B

Since this assumption can develop a one period pricing model only, the marked-to-the-market effect is thus neglected.

A2: Homogeneous belief on the spot price, $\tilde{S}(T)$ dividend payout, $\tilde{D}$, and index futures price, $\tilde{F}(T,T)$, is over all investors.

A3: Every investor maximizes his utility function which is $G_1(E_1, \sigma_1^2)$, where E_1 and σ_1^2 are the expected value and the variance of terminal wealth, respectively. It is at least second differentiable with respect to E_1 and σ_1^2 . The preference function, G_1 , is also an increasing function of E_1 and decreasing function of σ_1^2 . Furthermore, the marginal utility of the expected terminal wealth is decreasing

$$\left(\frac{\partial G_1}{\partial E_1} < 0 \right)$$

A4: The total units of demand in the spot market, X_s , is set in the form specified below for all the investors (N) in this market, namely,

$$\sum X_{si} = \sum S_{si} = X_s = X'_s \text{ (stands for gross supply).}$$

A5: The interest rate over the buying date to the maturity is treated as constant. Basically, it follows that the price of the future

is the same as the price of the forward contract (Cox, Ingersoll and Ross, 1981)

A6: No taxes, no transaction costs, price-taking investors, no indivisibility, and costless information are available to everyone.

Definitions

D1: $\tilde{S}(T) \equiv \tilde{I}(T) + \tilde{D}$ The index-equivalent portfolio, $S(t)$, is formed at time t . From time t to time T , this portfolio receives a total dividend payout, $\tilde{D}$. The value of the index-equivalent portfolio after dividend payout is $\tilde{I}(T)$ at time T . Thus, the index portfolio, $\tilde{S}(T)$, at time T is the sum of $\tilde{I}(T)$ and $\tilde{D}$.

D2: The risk tolerance of investor i is defined by

$$-\frac{\partial G_i / \partial E_i}{\partial G_i / \partial \sigma^2_i} \equiv \eta_i \quad \sum \eta_i \equiv \eta$$

where η is the total risk tolerance in economy.¹

D3: The percentage of the futures price change (or the "return") of the index futures in terms of index futures is

$$\tilde{r}_F \equiv \frac{\tilde{F}(T, T) - F(t, T)}{F(t, T)}$$

D4: The return of the spot index-equivalent portfolio

$$\tilde{r}_S \equiv \frac{\tilde{S}(T) - S(t)}{S(t)}$$

D5: The systematic risk of the index futures

$$\beta_F \equiv \frac{\text{Cov}(\tilde{r}_S, \tilde{r}_F)}{\text{Var}(\tilde{r}_S)}$$

D6: The percentage of the futures price change of the index futures in the terms of spot index is

$$\tilde{R} \equiv \frac{\tilde{F}(T,T) - F(t,T)}{I(t)}$$

D7: The hedge ratio of the index futures against the spot index is

$$\frac{B}{H} \equiv \frac{\text{Cov}(\tilde{F}(T,T), \tilde{S}(T))}{\text{Var}(\tilde{S}(T))} = \frac{\sigma_{SF}}{\sigma_S^2}$$

Model

This approach is to determine conditions for equilibrium of exchange of the three assets: the index-equilibrium portfolio, the index futures contract underlying this spot portfolio and the bond. Each individual brings to the market his or her present holdings of these three assets and an exchange takes place. This equilibrium approach needs to know what the price must be in order to satisfy demand schedules and also fulfill the condition that supply and demand be equal for all three assets. To answer this question, two requirements must be met. First relationship describing individual demand must be established. Second, these relations of all investors' demand are incorporated into a system to describe general equilibrium.

Assuming that there are a large number of risk averse investors labeled i ($i=1,2,\dots,N$), we can consider the behavior of a typical investor. He or she has to form a portfolio by choosing from the following three assets, namely, the index-equivalent portfolio, the index futures, and the risk-free bond. The future value on these three assets is assumed to be a random variable whose distribution is known to

the investor (Assumption A-1). Moreover, all investors are assumed to have identical perceptions of these probability distributions (Assumption A-2). The future value on a whole portfolio is, of course, a random variable. The portfolio analyses mentioned earlier assume that, in the choice from all the possible combinations, the investor is satisfied to be guided by its expected future value and its variance only (Assumption A-3).

It is important to make precise the description of a portfolio in these terms. It is obvious that the holdings of these three assets must be measured in some kind of units. Because the index futures contract is exactly a derivative asset under a particular index-equivalent portfolio, we select one "physical" unit of the spot portfolio as our measure unit and define expected future value and variance of future value relative to this unit.

It is convenient to give an interpretation to the concept of "future (dollar) value" by assuming discrete market dates with intervals of one time unit in an equilibrium perfect market (Assumption A4-A6). The future value to be considered on any asset on a given market date may then be thought of as the value per unit that the asset will have at the next market date (including accrued dividends).

The main purpose of this dissertation is to compare the relations between the price and the future value of the index-equivalent portfolio and the index futures. To facilitate such comparisons, a risk free bond is used as yardstick. Now, partial equilibrium conditions are capable of determining relative prices: to fix the index-equivalent portfolio's price and express the index futures on it.

The utility function of investor i is denoted as $G_i(E_i, \sigma_i^2)$. Likewise, the current value of investor i 's portfolio, or his budget constraint, is the following eq (3.1). Therefore, the terminal wealth of investor i 's portfolio is expressed in eq (3.2).

The Budget Constraint:

$$X_{si} \equiv W_{ti} = X_{si} I(t) + b_i \exp(-r_f) + O \times X_{fi} \dots\dots\dots(3.1)$$

The Terminal Wealth:

$$\tilde{W}_{Ti} = X_{si} \tilde{s}(T) + X_{fi} [\tilde{F}(T, T) - F(t, T)] + b_i \dots\dots\dots(3.2)$$

The expected terminal wealth and variance of the terminal wealth are listed as eq (3.3) and eq (3.4), respectively.

The Expected Terminal Wealth:

$$E_i \equiv E(\tilde{W}_{Ti}) = X_{si} \bar{S}(T) + X_{fi} [\bar{F}(T, T) - F(t, T)] + b_i \dots\dots\dots(3.3)$$

Variance of Terminal Wealth:

$$\begin{aligned} \sigma_i^2 &\equiv \text{Var}(\tilde{W}_{Ti}) \\ &= X_{si}^2 \sigma_s^2 + X_{fi}^2 \sigma_f^2 + 2X_{si} X_{fi} \sigma_{sF} \\ &= X_{si}^2 (\sigma_I^2 + 2\sigma_{ID} + \sigma_D^2) + X_{fi}^2 \sigma_F^2 + 2X_{si} X_{fi} \sigma_{IF} + 2X_{si} X_{fi} \sigma_{DF} \end{aligned} \dots\dots\dots(3.4)$$

Formally, then, we postulate that each investor i , who behaves as if attempting to maximize the utility function, is subject to the budget constraint, the expected value and variance of his portfolio to form his or her portfolio. Forming the LAGRANGEAN for each investor in eq (3.5) and differentiating with respect to X_{si} , X_{fi} , and b_i yields the first order conditions in eq (3.6) to eq (3.9).

The Lagrangean Form:

$$L_1 = G_1 (E_1, \sigma_1^2) + \lambda_1 (W_{t1} - X_{s1} I(t) - b_1 \exp(-r_f)) \dots\dots\dots(3.5)$$

Maximization:

$$\text{Max } L_1 \\ X_{s1}, X_{f1}, b_1 \quad \text{s.t. (3.3) and (3.4)}$$

First Order Condition²:

$$\frac{\partial L_1}{\partial X_{s1}} = \frac{\partial G_1}{\partial E_1} (\bar{I}(T) + \bar{D}) + \frac{\partial G_1}{\partial \sigma_1^2} [2X_{s1} \sigma_s^2 + 2X_{f1} \sigma_{sF}] - \lambda_1 I(t) = 0 \dots\dots\dots(3.6)$$

$$\frac{\partial L_1}{\partial X_{f1}} = \frac{\partial G_1}{\partial E_1} (\bar{F}(T, T) - F(t, T) + \frac{\partial G_1}{\partial \sigma^2} [2X_{f1} \sigma_F^2 + 2X_{s1} \sigma_{sF}] = 0 \dots\dots\dots(3.7)$$

$$\frac{\partial L_1}{\partial b_1} = \frac{\partial G_1}{\partial E_1} - \lambda_1 \exp(-r_f) = 0 \dots\dots\dots(3.8)$$

$$\frac{\partial L_1}{\partial \lambda_1} = W_{t1} - X_{s1} I(t) - b_1 \exp(-r_f) = 0 \dots\dots\dots(3.9)$$

These first-order conditions may be aggregated to derive the equilibrium relationship. Subtract eq (3.6) from eq (3.7), and rearranging, we obtain eq (3.10).

$$\begin{aligned} & \frac{\partial G_1}{\partial E_1} \{ [\bar{I}(T) + \bar{D}] - [\bar{F}(T, T) - F(t, T)] \} \\ & + \frac{\partial G_1}{\partial \sigma_1^2} 2[(\sigma_s^2 - \sigma_{sF}) X_{s1} + (\sigma_{sF} - \sigma_s^2) X_{f1}] - \lambda_1 s(t) = 0 \end{aligned} \dots\dots\dots(3.10)$$

Dividing both side of eq (3.10) yields eq (3.11)

$$\begin{aligned} & \frac{\partial G_1 / \partial E_1}{\partial G_1 / \partial \sigma_1^2} [\bar{I}(T) + \bar{D} - \bar{F}(T, T) + F(t, T)] + 2[X_{s1}(\sigma_s^2 - \sigma_{sF}) \\ & + X_{f1}(\sigma_{sF} - \sigma_F^2)] - \frac{\partial G_1 / \partial E_1}{\partial G_1 / \partial \sigma_1^2} \exp(r_f) I(t) = 0 \end{aligned} \quad \dots\dots\dots(3.11)$$

To obtain the pricing relationship, we have to aggregate over all investors.

To determine general equilibrium, we must also specify equality between demand and supply for each asset. Recall that X_{fi} , X_{si} , b_i were defined as the units of (net long) index futures, spot index-equivalent portfolio and the bond held by investor i . These market clearing conditions can be written as the following: $\sum X_{fi} = 0$, $\sum X_{si} = X_s$, $\sum b_i = B$. This essentially completes the equations describing a partial equilibrium. Also, in equilibrium, eq (3.11) must hold for all investors and if we assume that all investors have homogeneous expectations regarding E_1 and σ_1^2 we can sum eq (3.11) over i and define the risk tolerance, η , to arrive at the closed-form in eq (3.12).³

$$\begin{aligned} F(t, T) &= I(t) e^{r_f} + 2/\eta X_s (\sigma_s^2 - \sigma_{sF}) - [\bar{D} - \bar{I}(T) + \bar{F}(T, T)] \\ &= I(t) e^{r_f} + 2/\eta X_s (\sigma_s^2 - \sigma_{sF}) - \bar{D} \\ &= I(t) e^{r_f} + 2/\eta X_s \sigma_{DF} - \bar{D} \end{aligned} \quad \dots\dots\dots(3.12)$$

In eq (3.12), we have a term, $2X_s (\sigma_s^2 - \sigma_{sF})/\eta$, to measure the risk premium of the random dividend effect. The sign and value of this premium

is thus an empirical question. Dividing eq (3.6) and eq (3.7) by $\frac{\partial G_1}{\partial \sigma_1^2}$

yields eq (3.13) and eq (3.14), respectively.

$$I(t) \exp(r_f) - \bar{I}(T) - \bar{D} = 2/\eta X_s \sigma_s^2 \quad \dots\dots\dots(3.13)$$

$$F(t, T) - \bar{F}(T, T) = 2/\eta X_s \sigma_{sF} \quad \dots\dots\dots(3.14)$$

After rearrangement of eq (3.13) and eq (3.14), we have eq(3.15).

$$\begin{aligned}\bar{F}(T,T) - F(t,T) &= [\bar{I}(T) + \bar{D} - I(t) \exp(r_f)] \frac{\sigma_{sF}}{\sigma_s^2} \\ &= [\bar{S}(T) - I(t) \exp(r_f)] \frac{\sigma_{sF}}{\sigma_s^2} \dots\dots\dots(3.15)\end{aligned}$$

Both sides of eq (3.15) divided by $F(t,T)$ yields eq (3.16)

$$\begin{aligned}\bar{r}_F &= \frac{\sigma_{sF}}{\sigma_s^2} \left[\frac{\bar{S}(T) - I(t)}{I(t)} - (\exp(r_f) - 1) \right] \frac{I(t)}{F(t,T)} \\ &= \frac{\text{Cov}(\tilde{r}_s, \tilde{r}_F)}{\text{Var}(\tilde{r}_s)} [\bar{r}_s - r_f] \\ &= \beta_F (\bar{r}_s - r_f) \dots\dots\dots(3.16)\end{aligned}$$

Equation (3.16) is similar to the conventional CAPM which applies to an asset holding no initial positions. That is, the introduction of index futures market does not change the basic structure of capital asset pricing under conditions of uncertainty. Hereafter, eq (3.16) is referred as Like-CAPM model. $\text{COV}(\tilde{r}_s, \tilde{r}_F) / \text{VAR}(\tilde{r}_s)$ is the risk level of the index futures relative to the spot index equivalent portfolio. The covariance term, $\text{cov}(\tilde{r}_s, \tilde{r}_F)$, is the key element for the risk premium of the index futures contract. This should come as no surprise, since the index futures contracts do include dividend risk. Furthermore, due to no investment in the index futures at the beginning, we have no intercept term. However, if we take margin into account, we should have the full CAPM. This is to say that the margin is treated as the performance bond.

Now, let us examine the conventional hedging strategy: regress the index futures price change on the spot index change to yield the hedge ratio subject to a holding period as the price change duration. After dividing eq (3.15) by $I(t)$, we arrive at eq (3.17).

$$\begin{aligned}\bar{R}_F &= \frac{\sigma_{sF}}{\sigma_s^2} (\bar{r}_s - r_f) \\ &= \beta_H (\bar{r}_s - r_f) \dots\dots\dots(3.17)\end{aligned}$$

With a position of long the spot index-equivalent portfolio and short the index futures at time t , using $1/\beta_H$ as the hedge ratio, we would have the risk-free rate of return.

In the next section, we intend to demonstrate a comparative statics analysis of this model, to explain who are going to short or long.

Comparative Statics Analysis

The comparative statics of security risk premium in a mean-variance context has received rigorous treatment in the literature.⁴ Including the index futures market, the comparative statics analysis of market equilibrium is not examined since the equilibrium price of index futures not yet thoroughly understood. We are interested in the factors affecting the optimal number of index futures contract. In this section, the types of comparative statics changes considered include the spot index futures price change, spot price change, the interest rate change, the expected dividend yield change, the expected basis change and the relative risk aversion measurement change. Statements thus are made concerning the expected effect on the demand of assets of an increase in these factors.

To facilitate a comparative statics analysis, we need to develop the optimal units of index futures and index-equivalent portfolio held by the investors.

Dividing eq(3.6) and eq(3.7) by $\frac{\partial G_1}{\partial \sigma_1^2}$, together with the budget constraint,

we have the optimal set as eq (3.18).

$$\begin{cases} X_{si}^* \sigma_s^2 + X_{fi}^* \sigma_{sF} = \eta_i/2 [\bar{I}(T) + \bar{D} - I(t) \exp(r_f)] \\ X_{si}^* \sigma_{sF} + X_{fi}^* \sigma_F^2 = \eta_i/2 [\bar{F}(T,T) - F(t,T)] \\ b_i^* = e^{rf} (W_{ti} - X_{si}^* s(t)) \end{cases} \dots\dots\dots(3.18)$$

Rearranging, we find the optimal holdings to be eq (3.19)

$$\begin{cases} X_{si}^* = \frac{[\bar{I}(T) + \bar{D} - I(t) e^{rf}] \sigma_F^2 - [\bar{F}(T,T) - F(t,T)] \sigma_{sF}}{Z} \\ X_{fi}^* = \frac{[\bar{F}(T,T) - F(t,T)] \sigma_s^2 - [\bar{I}(T) + \bar{D} - I(t) \exp(r_f)] \sigma_{sF}}{Z} \\ b_i^* = e^{rf}(W_{ti} - X_{si}^* s(t)) \end{cases} \dots\dots\dots(3.19)$$

where $Z \equiv 2/\eta_i (\sigma_s^2 \sigma_F^2 - \sigma_{sF}^2)$

These are functions of index functions price, spot index price, interest rate, dividend payout, degree of risk aversion, etc. Then we do some partial derivatives to examine factors which might affect the optimal hold units of index futures.

For risk adverse investors (which we assume), the risk-aversion measure, η_i , is positive. Therefore, Z , defined as $2(\sigma_s^2 \sigma_F^2 - \sigma_{sF}^2)/\eta_i$, is positive also. Thus, we have the following results, eq (3.20) to eq(3.26).

$$\frac{\partial X_{fi}}{\partial F(t,T)} = \frac{-\sigma_s^2}{Z} < 0 \quad \dots(3.20)$$

$$\frac{\partial X_{f1}}{\partial r_f} = \frac{I(t) \exp(r_f) \sigma_{sF}}{Z} > 0 \quad \text{.....(3.21)}$$

$$\frac{\partial X_{f1}}{\partial s(t)} = \frac{\exp(r_f) \sigma_{sF}}{Z} > 0 \quad \text{.....(3.22)}$$

$$\frac{\partial X_{f1}}{\partial D} = \frac{-\sigma_{sF}}{Z} < 0 \quad \text{.....(3.23)}$$

$$\frac{\partial X_{f1}}{\partial F(T,T)} = \frac{-\sigma_s^2}{Z} > 0 \quad \text{.....(3.24)}$$

$$\begin{aligned} \frac{\partial X_{f1}}{\partial \eta_1} &= \frac{[\bar{F}(T,T) - F(t,T)] \sigma_s^2 - [\bar{I}(T) + \bar{D} - I(t) \exp(r_f)] \sigma_{sF}}{Z} \\ &> 0 \quad \text{if numerator} > 0 \\ &< 0 \quad \text{if numerator} < 0 \end{aligned} \quad \text{.....(3.25)}$$

$$\frac{\partial F(t,T)}{\partial \sigma_{sD}} = \frac{2}{\eta_1} X_s > 0 \quad \text{.....(3.26)}$$

A model of demand or supply for the index futures and index-equivalent portfolio in equilibrium stage is shown as eq (3.19). These results are based on previous closed-form, pertaining to some restricted assumptions. Here, the optimal number of units of the index futures is derived first, assumed to be net long if positive. Equation (3.20) shows the law of demand in the index futures. The higher the index futures price is, the less one goes long the index futures. The index futures is a normal goods. The impact of the interest rate on the number of units of the index futures is shown in eq(3.21). The higher the interest rate is, the more the long position in the index futures would be. This result means that the index futures contract tends to be a free good if the interest

rate is very high. Furthermore, equation (3.22) demonstrates the relationship between holding the index-equivalent portfolio being long the index futures. The higher the spot index is, the more to long the index futures would be. So, these two financial products are substitute goods. Equation (3.23) shows the impact of dividend payout on the number of units to long index futures. If the expected dividend increases, the demand to long the index futures decreases. In equation (3.24), it is shown that expected maturity futures price affects the number of units to long the index futures. The higher the expected maturity futures price (or the expected profit to long the index futures) is, the more to long the index futures would be, other factors being constant.

According to the budget constraint and eq (3.25), individual initial wealth affects the investment decision through the combined effect of the degree of the risk-aversion, the expected price and the variance of the index futures price, and the covariance of index-equivalent portfolio price and index futures price. With regard to the performance bond, the performance bond holding is not influenced by the expectation of the index futures contract. It might suggest that bond holding is a supplementary good of the index futures market. In addition, the initial wealth does not directly affect the holding of these assets.

Comparative statics indicates that $\frac{\partial F(t,T)}{\partial \sigma_{SD}} > 0$. Thus the index

futures price could increase, according to eq (3.26), when the underlying index-equivalent portfolio dividend yield is stochastic if the covariance of future value on the index-equivalent portfolio and the dividend payout increases. This is due to the covariance of dividend payout and the index-equivalent portfolio being equal to the variance of dividend payout

plus the covariance of dividend payout and the futures price. (i.e., $\text{Cov}(\tilde{D}, \tilde{S}(T)) = \text{Var}(\tilde{D}) + \text{Cov}(\tilde{F}(T,T), \tilde{S}(T))$). Therefore, if dividend payout increases following the increase of the index-equivalent portfolio, then the index value would also increase following the increase of dividend payout. This means that the index futures price would increase too. Thus, the higher the correlation between the index-equivalent portfolio and dividend payout, the higher the index futures price would be. The index futures price is influenced by the covariance term between dividend payout and index-equivalent portfolio. Intuitively, this has to do with the fact that the index futures price still has to face the dividend risk even if the dividend payout does not benefit the individual who trades in the index futures contracts in long side or in short side.

Summary

We have presented a closed form model for index futures pricing in a partial equilibrium approach. When we allow for the existence of random dividend yield, this model does bear a risk premium for this random term. A covariance term of dividend payout and index-equivalent portfolio is added to the perfect market model. We can see that to evaluate the index futures contract is the same as a project evaluation. Market equilibrium causes the inflows received from being long a unit of index futures contract, (holding to the maturity i.e., $\tilde{F}(T,T) - F(t,T)$), to be inflows from borrowing fund to purchase a unit of index-equivalent portfolio (valued as \$500 times spot index) and hold this to the maturity of its corresponding index futures contract, over this period to collect the expected dividend yield and its associated dividend risk premium (i.e., $\tilde{I}(T) - I(t)e^{rf} + \bar{D} - 2X_S \text{Cov}(\tilde{D}, \tilde{S}(T))/\eta$).

It means that the cash inflow to long an index futures contract is quite similar to implementing a capital investment project with borrowing funds, over the holding period to collect its profit or loss which is marked to the market and including the uncertain dividend payout or earning performance. Meanwhile, with comparative analysis, we know the index futures is a normal good. The demand to be long index futures increases, if the interest rate, the index-equivalent portfolio price, the expectation of the future spot price, or the correlation between index-equivalent portfolio and dividend payout increases. On the contrary, the demand to be long the index futures decreases if expected dividend decreases. Moreover, the more risk-adverse the investor is, the less the demand to long the index futures will be.

When we express this model in rate of return terms, somewhat surprisingly, the equilibrium market relationship between risk and "market return" on index futures contract is still of the same general linear form as that of the Sharp-Lintner-Mossin model. The primary testable implications of the model are that the linearity of the relationship between risk and "expected return," or the "hedge ratio" is greater than one, or the dividend risk premium exists in the market.

Notes

- [1] If preference function is a quadratic function, then this measure is identical to risk tolerance defined by Pratt (1964). Expected quadratic utility function is only a special function of $G_i(E_i, \sigma_i^2)$.
- [2] Without loss of generality, the utility function satisfies some other condition such that second order condition of optimality is satisfied.
- [3] It is consistent with the Ederington (1979) hedge argument. If dividend payout is a random term indeed, the hedge ratio is not equal to one as the traditionalists argued. Furthermore, assuming that the market setting as follows:

Market	Price at t	Future Value at t+1	# held	Aggregate
Spot Market	$S(t)$	$S(t+1)$	X_s	X
Index Futures	0	$\tilde{F}(t+1, T) - F(t, T)$	X_{fi}	0^s
Bond	$\exp(-r_f)$	1	b_i	B

then we could have the pricing relation as eq (3.12.1)

$$F(t, T) = I(t)e^{rf} + 2/\eta X_s [\text{Var}(\tilde{S}(t+1)) - \text{Cov}(\tilde{S}(t+1), \tilde{F}(t+1, T))] \\ + [-\bar{D}_{t, t+1} - \bar{F}_t(t+1, T) - \bar{I}_t(t+1)] \dots\dots\dots (3.12.1)$$

where $\bar{D}_{t, t+1}$, $\bar{F}_t$, $\bar{I}_t$ are expected value of $\tilde{D}$, $\tilde{F}$, $\tilde{I}$ based on the information set at time t.

With arrangement, it becomes eq (3.12.2)

$$[F(t, T) - \bar{F}_t(t+1, T)] - [\bar{I}(t) - \bar{I}_t(t+1, T)] \\ = \bar{D}_{t, t+1} + I(t)(e^{rf} - 1) + 2/\eta X_s [\text{Var}(\tilde{S}(t+1)) - \text{Cov}(\tilde{S}(t+1), \tilde{F}(t+1, T))] \\ \dots\dots\dots (3.12.2)$$

It shows the equilibrium basis relationship. The aggregate basis risk thus is affected by the interest rate, the dividend, the risk aversion in the economy, the market return, the total units of index-equivalent portfolio in the capital market, and the holding period.

- [4] The analysis of the spot market is reported by others in detail. So, we only analyze the index futures case.

CHAPTER IV THE EMPIRICAL STUDY

In chapter III, the theoretical price of an index futures contract using an equilibrium model was stated in eq(3.12). A major difference between this model and those based upon an arbitrage approach is the potential importance of an unknown dividend. A covariance term between the dividend payout and the index-equivalent portfolio is added to the equilibrium pricing model. In the arbitrage model, the dividend payout is assumed to be known in advance. In this chapter, we investigate whether this covariance term empirically appears to exist in the actual trading market.

The dividend risk argument of this dissertation is contingent on the existence of a covariance between dividend payouts and an index-equivalent portfolio. We have also linked the theoretical index price using the equilibrium approach to the conventional CAPM model and a hedging relationship as shown in eq(3.16) and eq(3.17), respectively. If dividend risk is important in index futures pricing, the Like-CAPM model, i.e., eq(3.16), suggests a null intercept as well as a systematic risk level greater than one. Therefore, the Like-CAPM model can serve as a supportive test. If dividend risk is present and priced in the markets, the Like-CAPM model predicts that the systematic risk is greater than one. In addition, the existence of a non-zero intercept in the Like-CAPM model is important in that it is suggestive of another missing variable, such as the margin requirement. If the intercept of the regression is not significantly

different from zero, we have no evidence that this is an important factor in pricing index futures. Similarly, a conventional hedge relationship between the spot and the index futures is derived, as shown in eq(3.17). This hedge relationship also serves as a supportive test to the dividend risk argument: the slope of the regression is expected to be greater than one if dividend risk exists in the market.

These three equations, eq(3.12), eq(3.16) and eq(3.17), serve as empirical devices to evaluate the impact of dividend risk and to describe the behavior of any such dividend risk premium. The pricing equations are, strictly speaking, correct only in a one period case. This means that the futures contract is being treated as a forward contract.¹

The dividend risk argument has two critical considerations which must be understood. First, when a dividend payment is declared by a company, there is no dividend risk present any longer between the date of the announcement and subsequent payment. However, for the index-equivalent portfolio, it is not easy to eliminate total dividend risk at any specified date, since not all companies in the index-equivalent portfolio announce dividend payments at the same date. A varying degree of dividend risk is therefore expected to exist in the market. The nearer the maturity date of the index futures contract is approaching, the less the corresponding dividend risk would probably be. Therefore, the last several trading days might bear insignificant dividend risk. Second, most companies pay dividends quarterly which causes a seasonality in dividend payout behavior. Since the dividends of most companies are declared before paid, the dividend risk premium of the index futures price might be affected by the behavior of the dividend payout pattern of the spot index.

For these two reasons, we report two tables of dividend payout distributions. The first table (Table 4.1) describes the frequency of the number of days between the date of a dividend announcement and the eventual ex-dividend date for all companies listed in the S&P 500 and the NYSE. This table reveals the degree of certainty about future dividend payments at points in time prior to when the dividend actually is paid. The second table (Table 4.2) describes the monthly payout pattern during the years 1981-1983. These two tables motivate the basic empirical design of this dissertation.

In what follows, we discuss first how the preliminary results in Table 4.1 and 4.2 were obtained. Following that variable measurement, test methodology, test hypotheses and data set problems are discussed. The main content of this second section is to examine the difficulty of using a direct testing methodology, and, in its place develop a dummy variable approach for indirect testing. In the third section, results of a stricter test methodology as well as tests of the Like-CAPM and the hedging relationship are evaluated. Finally, a brief summary is provided in the fourth section.

A Preliminary Result

In this section, two tables are presented to highlight the empirical study: (1) a table regarding the frequency of dividend events and (2) a table regarding the seasonality of monthly dividend payments.

Based on the period 1982-1983, the dividend data on the CRSP tape was used to evaluate the degree of dividend certainty over different

lengths of time. This is done by noting the number of days which transpire between the date of a dividend declaration and its eventual payment (actually, the ex-dividend date.) This was done for (1) all companies included in the S&P 500 and (2) all companies listed on the NYSE. These two indices were chosen since it is exactly these indices on which the principle stock index futures are written.

The results are reported in Table 4.1. Notice that seventy-five percent of the observations had announcement dates within thirty days from the ex-dividend date. This is true for both the NYSE and S&P 500. It suggests that dividend risk is not present in the period up to thirty days before the ex-dividend date for seventy five percent of all firms. Thus, trading in the index-equivalent portfolio and the index futures bear substantially less dividend risk within the thirty days before ex-dividend date.

In addition, a monthly dividend payout pattern (from 1981 to 1983) is displayed in Table 4.2. There are four peak dividend payout months. They are February, May, August, and November.

These results suggests that the behavior of dividend risk may be characterized by seasonality and by the length of time before the contract matures.

Basic Discussion

Direct testing of eq(3.12) requires data on futures prices, interest rates, spot price of the index, an expected dividend, and the covariance between dividend payout and the index-equivalent portfolio. Of these, the futures price, spot price, and interest rate are directly observable. The expected dividend could be estimated in a number of

ways. Unfortunately, the covariance term is a cross section term and presents a severe estimation problem. To compute the covariance between the dividend payout and the index-equivalent portfolio requires that we specify the holding period being considered. One can not simply examine the historical covariance between, say, one day spot price changes and dividends paid on that date. First, the dividends would be known on that date since they would have been previously declared. Second, this would focus only on a one day holding period. It would not begin to evaluate the covariance when the holding period is different. And, if we wish to evaluate holding periods longer than one day, it is almost impossible to specify which holding periods should be examined. Directly testing is quite difficult. We turn, instead, to an indirect test which uses a dummy variable approach.

But, it is still necessary to estimate the expected dividend payout per unit for the index-equivalent portfolio. Denoting $\bar{D}$ as the expected dividend, the estimation method used is as follows:

$$\bar{D} = \frac{\text{Accumulated Dividend of the index from } t \text{ to } T}{\text{Market Value of the index at } t} \times \frac{S(t)}{500} \dots\dots\dots(4.1)$$

In addition, we set up a dummy variable approach to represent the covariance term over the holding period. This is done by using the additive property of the covariance term to handle the holding period problem. The dummy variable coding is set to a specified value in an attempt to measure the holding period effect of dividend risk. Two approaches are used. First, a twelve-calendar-month dummy variable approach captures the monthly effect. Second, an approach based on a three-month payout pattern is used to capture any seasonal effect in the dividend risk premium.

Equipped with an estimate of the expected dividend, ($\bar{D}$ as defined above) and the number of index-equivalent portfolio, ($I(t)$) in the market, eq(3.12) can be empirically tested as a regression of $F(t,T)$ (adjusted by the expected dividend of index-equivalent) on the future certainty-equivalent portfolio, and the twelve dummy variables. However, to get a conventional R-square, we need to subtract from the right hand side the future certainty-equivalent portfolio. This results in a new dependent variable, which we call the "futures discount." This "futures discount" is regressed on the various dummy variables.

Using this model, we might still have two problems. First, a serial correlation might arise. Second, we have to consider the stationarity of the residuals. In order to assure that the regression residuals are stationary, the regression should be performed on a rate of change or on a generalized difference price change. In fact, it is the rate of change in the futures price, not the price level alone, that is the variance of economic interest. Taking generalized first differences of eq(3.12), we solve the serial residual problem and the serial correlation adjustment could provide a more efficient testing.

In summation, the generalized difference equation states that the change in the index futures price, adjusted for both the expected dividend and the value of the future certainty-equivalent of spot portfolio, is equal to the dividend risk premium as captured by various dummy variables.

Data Sets

Following the preliminary result above, five formal tests are motivated: (1) the use of a calendar-month dummy variable approach (2) the

use of seasonal dividend dummy variable approach (3) a test of the market pricing model, (4) a test of the Like-CAPM relationship, and (5) a test of the hedge relationship.

These five tests are performed on a data base of the two most commonly traded indices: the S&P 500, and the NYSE. There are six index futures simultaneously traded in the futures market, since every futures contract lasts eighteen months (except in the case of earliest contracts offered) and new contracts have maturity dates which are three months apart. For all futures contracts, daily closing prices (at 4:15 P.M. Chicago Time) are obtained. For the spots, daily closing prices are also observed (at 4:00 P.M. Chicago Time). The Treasury Bill futures series is used as an interest rate series. The sample period is from the first trading date of every futures to March 8, 1984. In all cases, the percentage price change (or the "return") of the index futures is computed as a one-day holding. In addition, the dividend payout is assumed to be reinvested at the risk-free rate daily during the holding period. Since all subsequent statistical measures are assumed to be serially independent price changes, serial correlation coefficients of order one will be computed for each futures price series in order to adjust to the fact that price tend to move together. Since we use generalized first difference least squares to obtain efficient estimators, all observations which involve Monday or the day after a holiday are excluded in order to avoid a different time span problem.

Unexpectedly, the dividend data series is not easily accessible. For New York Index Futures, the daily dividend payout series of the unit index-equivalent portfolio is only available from January 1, 1983.

This was provided by the NYSE statistics department. For the S&P 500, data is aggregated from the CRSP daily tape (from Center Research of Security Prices, University of Chicago) based on the December 1982 COMPUSTAT TAPE (from Standard & Poor's Computing Service, Inc) cusip list. Unfortunately, 20 over-the-counter companies are not accessible from the CRSP tape and data from 13 other companies are not fully covered over the sample period. Therefore, 6.6% of the S&P 500 is missing in the data set, meaning that we only create an S&P 467 dividend and market value data set. In terms of both the dividend and market value on December 1982, this S&P 467 dividend payout data series underestimates the total S&P 467 dividend payout data series the total S&P 500 by 6.6%. What we need, however, is the dividend payout per unit of index futures. As a consequence, we divide the total dividend by the market value to get the dividend per unit. This "unit" dividend is still approximately acceptable. The effective sample period is from January 1, 1983 to December 31, 1983 for the NYSE Index Futures and from April 21, 1982 to December 31, 1983 for the S&P 500 Index Futures. The data base is listed in Appendix A.

Seven data subsets are named and tested for each contract. They are follows:

1. WHOLE: all observations of the index futures contracts.
2. LE30: all observations of the index futures contracts, excluding the last thirty trading-day observations.
3. LAST30: the last thirty trading-day observations of the index futures contract.
4. LAST90: the last ninety trading-day observations of the index futures contracts.

5. LE90: all observations on the index futures contracts, excluding the last ninety trading-day observations.
6. L120LE30: the last one hundred and twenty observations, excluding the last thirty trading-day observations.
7. LE120: all observations of the index futures contracts, excluding the last one hundred and twenty trading-day observations.

The reason for forming these data sets is that they enable us to analyze the dividend risk in different respects. In the first place, WHOLE is necessary for us to have a whole picture of this "futures discount." Second, we need to know what happens to the nearest (to the maturity) contract. Therefore, LAST90 is useful. Third, the trading in the maturity month should have minimal dividend risk exposure. Thus, LAST30 is needed in order for us to see whether the perfect market model holds. Others are examined to see if there is a change in the pattern of dividend risk over different periods.

Test Methodology and Results

In the previous section, five tests related to three equations, eq(3.12), eq(3.16), and eq(3.17), were briefly discussed. In this section, the test methodology of each is presented in more detail. Results are reviewed in the subsequent subsections. The first subsection discusses the use of a calendar-month dummy variable approach. The second subsection discusses the use of a dividend-payout-pattern dummy variable approach. The above two tests methods essentially employ the same concept except that they use a different dummy variable coding to capture the dividend risk premium. Based on initial results,

the last thirty trading days before the index futures mature might not be exposed to dividend risk. If so, the perfect market model should hold in LAST30 data set. Therefore, the test of the perfect market model is performed in the third subsection. Other related issues, such as the test of Like-CAPM and the test of the hedge relationship, are discussed in the fourth subsection as supportive tests of the dividend risk argument. Finally, the fifth subsection summarizes the findings of this chapter.

Use of a Calendar-Month Dummy Variable Approach

In eq(3.12), the dividend risk adjustment is contingent on the existence of a covariance term between dividend payout and the index-equivalent portfolio. The dividend payout referred to is the accumulated daily dividend payment over the holding period. Assume that the dividend accumulating process is as follows:

$$\tilde{D} = \sum_{i=0}^{T-t-1} \tilde{D}_{t+i,t+i+1} \quad \dots\dots\dots(4.3)$$

Since the covariance has an additive property, we have the following equation.

$$\text{Cov}(\tilde{D}, \tilde{S}(T)) = \sum_{i=0}^{T-t-1} \text{Cov}(\tilde{D}_{t+i,t+i+1}, \tilde{S}(T)) \quad \dots\dots\dots(4.3)$$

The covariance term which is of interest depends upon two things:

- (1) individual covariance over shorter period of time (say, monthly or seasonal) and
- (2) the number of such periods which will occur from the date of pricing to maturity, i.e., time.

To capture this covariance relationship, we will first use a monthly dummy variable approach as follows. Define the dummy variable coding, M_i , $i=1,2,\dots,12$, for the dividend risk premium of each calendar month.

Values of any M_i can vary from zero to two, depending on how many times the calendar month is duplicated over the holding period. For example, if an investor buys or sells a unit of index futures and holds from t to T , then the covered calendar-month dummy variable of the twelve dummy variables is valued as one or two depending on whether the months are duplicated one or two times. Other non-covered calendar month dummy variables are set to be zero. For example, if we buy a December 1984 contract in June 1983, then the dummy variables standing for June through December are two and others are one. Similarly, if it is bought in June 1984, then the dummy variables standing for June through December are one and others are zero.

To facilitate the statement of empirically testable null hypotheses, the ex ante equilibrium relation can be restated as an ex post relation assuming that (1) the independent variables contain no measurement errors, (2) that they are stationary, and (3) that individual expectations defined over the variates are unbiased. For OLS estimators to be efficient, it is required that observations be serially uncorrelated. Unfortunately, serial correlation is usually present in most time series regression in the first step of the estimation process. In our test, if serial correlation occurs, the Durbin procedure is used to adjust the ordinary least-squares regression procedure to obtain efficient parameter estimates. This is referred to here as the second step of the estimation process. This procedure involves the use of generalized differencing to alter the linear model into one in which the errors are independent. Economically, it implies that the prior price information is carried over to today by the degree of serial correlation. Estimation of this generalized differencing results in a new, more efficient, set of parameter estimates.

Since the empirical model consists of the index futures regressed on the future certainty-equivalent portfolio, and the twelve calendar-month dummy variables without an intercept term, the R-square would be almost invariably be one. This is due to the exclusion of the intercept in the regression. Given this, a new dependent variable, which has been labeled futures discount, is created. Futures discounts is defined to be the value of the index futures price after subtracting the future certainty-equivalent spot portfolio and adding back the expected accumulated dividend payout per unit. In addition, the future certainty-equivalent spot portfolio must be an independent variable in the regression. Hereafter, we use FD and FCESP to indicate futures discount and the future certainty-equivalent spot portfolio, respectively.

To summarize, a regression of FD on $\underline{M}$ and FCESP is performed.² The results of the first step regression are reported in Appendix B. The results of the second step regression are interpreted after the mathematical details of this test are examined.

Test I: Test of eq(3.12) using monthly dummy variable approach

Step 1: The first step regression

$$\text{Model: } FD(W) = \alpha \underline{M}(W) + \beta FCESP(W) + \epsilon_w \quad \dots\dots(4.4)$$

where

$$1) \quad FD(W) \equiv F(W,T) + \sum_{i=W}^T D(i) \exp(r_f (T-i)/(T-t)) \\ - I(W) \exp(r_f (T-W)/(T-t))$$

$$2) \quad \underline{M}(W) \equiv \begin{Bmatrix} M_1(W) \\ M_2(W) \\ \vdots \\ M_{12}(W) \end{Bmatrix} \quad M_1(W) = \begin{cases} 0 \\ 1 \\ 2 \end{cases} \\ i=1,2,\dots,12$$

$$3) \quad FCESP \equiv I(W) \exp(r_f (T-W)/(T-t))$$

$$4) \quad \epsilon_w = \rho \epsilon_{w-1} + \xi_w$$

$$5) \quad t < W < T$$

THE NULL HYPOTHESES:

$$(1) \quad \underline{\alpha} = 0$$

$$(2) \quad \beta = 0$$

THE ALTERNATIVE HYPOTHESES:

$$(1) \quad \underline{\alpha} \neq 0$$

$$(2) \quad \beta \neq 0$$

These estimators, however, are not efficient due to serial correlation in the error term. To obtain an efficient estimator, a second step regression using the serial correlation estimate of the first step regression, $\hat{\rho}$, is performed.

Step 2: The second step regression

$$\text{Model:} \quad \Delta FD(W) = \underline{\alpha} \Delta \underline{M}(W) + \beta \Delta FCESP(W) + \xi_w \quad \dots(4.5)$$

where

$$1) \quad \Delta FD(W) = FD(W) - \hat{\rho} FD(W-1)$$

$$2) \quad \Delta \underline{M}(W) = \underline{M}(W) - \hat{\rho} \underline{M}(W-1)$$

$$3) \quad \Delta FCESP(W) = FCESP(W) - \hat{\rho} FCESP(W-1)$$

$$4) \quad t < W < T$$

THE NULL HYPOTHESES:

$$(1) \quad \underline{\alpha} = 0$$

$$(2) \quad \beta = 0$$

THE ALTERNATIVE HYPOTHESES:

$$(1) \quad \underline{\alpha} \neq 0$$

$$(2) \quad \beta \neq 0$$

Note that this linear transformation does not change the null and alternative hypotheses.

Results of step one on the data sets of WHOLE and LE30 are reported

in Appendix B, Table B.1-B.2, respectively. Results of step two are in Table 4.3 and Table 4.4 for data sets of WHOLE and LE30.

We find that the slope coefficients of the calendar-month dummy variables are mostly negative with significance at 1% significance level. Not surprisingly, the absolute value of the slopes are larger in the data set LE30 than they are in the data set WHOLE (which includes the last month of trading.) This implies that the earlier the transaction is, the larger the futures discount will be. Intuitively, the accumulated dividend risk (i.e., the sum of the coefficients of several calendar-month dummy variables) is a potential factor which would cause this discount to decrease over time. It suggests that a transaction in the early trading days of contract is riskier due to the higher uncertainty of total dividend payout. If dividend uncertainty is reduced, such as at the time when many companies submit their 10-K form and annual report around February, the dividend risk is not a significant factor in creating futures discounts any more. In some cases, the March dummy variable is positive with significance (at 1% significance level.) This positive effect might be caused by any number of things, a change in the investment opportunity set, for example. Occasionally, multicollinearity does occur in some regressions. This is caused by the fact that the observations all duplicate at the maturity (calendar) month, such as L120Le30 consisting of maturity month observations, or the effective observations of some months are missing due to no active trading in that month. In this case, we simply drop one variable which is collinear to another one. The slope of the remaining variable in the regression would then be the sum of the slopes of the two which are multicollinear before we drop it. At any rate, it does not affect

results and their implications. There is evidence that dividend risk is priced in the market. Meanwhile, since all observations always duplicate at the maturity month, the calendar-month dummy variable, which also represents the maturity month, would have a larger coefficient. This larger value actually is mixed by the true value with the intercept.³

Use of a Dividend-Payout-Pattern Dummy Variable Approach

The results of the monthly dummy variable approach discussed above are consistent with the possibility of a dividend risk premium. They are, however, consistent with other explanations. For example, simply time alone (the greater the time to expiration the greater the discount due to whatever reason) could be the cause of these results. The only part of the tests which points strongly towards an exclusion dividend risk impact is the insignificance of February, a month in which corporate reports would tend to reduce risk. Yet the financial literature has not observed a strong February effect on other areas of risk measurement.

What is needed is a test which is more closely associated with dividends themselves, a test which confirms or rejects solely the dividend risk argument. That is precisely what the tests associated with seasonal dummies are designed to do.

In Table 4.2, a dividend payout pattern was seen with peak months in February, May, August, and November. Three dummy variables are defined in order to investigate the possible seasonality of dividend risk premium. These three dummy variables are called the dividend-payout-pattern dummy variables, S_0 , S_1 , and S_2 . Their corresponding values are set as follows.

$$S_0 = M_2 + M_5 + M_8 + M_{11}$$

$$S_1 = M_1 + M_4 + M_7 + M_{10}$$

$$S_0 = M_{12} + M_3 + M_6 + M_9$$

Since a futures contract is eighteen months forward, the value of the dividend-payout-pattern dummy variables can take on values between one and six.

The basic idea underlying this approach is to examine whether a seasonality of the futures discount appears corresponding to the dividend payout pattern. In other words, we want to investigate the behavior of the dividend risk premium. In spite of this objective, the behavior of dividend risk might have other complications. The changing pattern along with its magnitude could be further estimated. The test procedure is the same as the first subsection. The mathematical details and hypotheses are as follows.

Test II: Test of eq(3.12) in a dividend-payout-pattern dummy variable approach

Step 1: The first step regression

$$\text{Model: } FD(W) = \alpha \underline{S}(W) + \beta \text{FCESP}(W) + \epsilon_w \quad \dots\dots(4.6)$$

where

$$1) \quad FD(W) \equiv F(W, T) + \sum_{i=W}^T D(i) \exp(r_f (T-i)/(T-t)) \\ - I(W) \exp(r_f (T-W)/(T-t))$$

$$2) \quad \underline{S}(W) = \begin{bmatrix} \text{two month before dividend-peak month} \\ \text{one month before dividend-peak-month} \\ \text{during dividend-peak-month} \end{bmatrix} \\ = \begin{bmatrix} S_2(W) \\ S_1(W) \\ S_0(W) \end{bmatrix}$$

$$S_i(W) = \begin{cases} 1 \\ 2 \\ \cdot \\ \cdot \\ 6 \end{cases} \quad i = 0, 1, 2$$

$$3) \text{ FCESP}(W) \equiv I(W) \exp (r_f (T-W)/(T-t))$$

$$4) \varepsilon_w = \rho \varepsilon_{w-1} + \xi_w$$

$$5) t < w < T$$

THE NULL HYPOTHESES:

$$(1) \underline{\alpha} = 0$$

$$(2) \beta = 0$$

THE ALTERNATIVE HYPOTHESES:

$$(1) \underline{\alpha} \neq 0$$

$$(2) \beta \neq 0$$

Step 2: The second step regression

$$\text{Model: } \Delta \text{ FD}(W) = \underline{\alpha} \Delta \underline{S}(W) + \beta \text{ FCESP}(W) + \xi_w \quad \dots\dots\dots(4.4)$$

where

$$1) \Delta \text{ FD}(W) = \text{FD}(W) - \hat{\rho} \text{ FD}(W-1)$$

$$2) \Delta \underline{S}(W) = \underline{S}(W) - \hat{\rho} \underline{S}(W-1)$$

$$3) \Delta \text{ FCESP}(W) = \text{FCESP}(W) - \hat{\rho} \text{ FCESP}(W-1)$$

$$4) t < W < T$$

THE NULL HYPOTHESES:

$$(1) \underline{\alpha} = 0$$

$$(2) \beta = 0$$

THE ALTERNATIVE HYPOTHESES:

$$(1) \underline{\alpha} \neq 0$$

$$(2) \beta \neq 0$$

The results summarized are derived after performing step one and step two on WHOLE, LE30, LAST30, L120LE30, LE90, LE120 data sets, respectively. Some of the results are quite significant. The results of the first step regression are presented in Appendix B, Table B.3-B.5, respectively. The results of the second step regression are

reported in Table 4.5, Table 4.6, and Table 4.7.

Before examining the results, a review of the test's logic is appropriate. Suppose a quarterly dividend payout is just announced, the dividend risk should be partially or temporarily reduced. Will the announcement affect the structure of the dividend risk premium? This hypothesis expands on the results of Test I (monthly dummies.) The extent to which uncertainty is resolved by the announcement can be understood by examining the trend of the slope coefficient of dividend-payout-pattern dummy variables. If the announcement essentially resolves all uncertainty, the dividend risk premium would be zero. On the other hand, the announcement could represent partial resolution only.

In Table 4.5, Table 4.6 and Table 4.7, we find that the risk premium of two months before and one month before are significant over the following data subsets: WHOLE, LE120, LE90. However, the risk premium of the dividend-peak-payout month is not significantly different from zero. Furthermore, the observations of two months before the dividend peak month bear the sum of the dividend risk premium of two months before and one month before dividend-peak-month, suggesting that dividend risk decreases when the dividend announcement is forthcoming. Finally, the uncertainty is resolved at the announcement. These results confirm the earlier conclusion that the futures discount can be explained by a factor which has seasonal pattern. Furthermore, since we are also interested in the changing pattern of dividend risk of the nearest contract, we apply the regression on the L120LE30 and LAST90. As shown in Table 4.7, only two months before the maturity is significant (at 1%

significance level). Recall that Table 4.1 shows that 98 percent of dividend events leave no dividend uncertainty within sixty trading-day of the ex-dividend date. Therefore, the fact that there is no significance in both one month before and during the dividend-peak-month dummy variables suggests strongly that the dividend risk is reduced when time to the maturity is within sixty days before dividend announcement. The dividend risk indeed disappears in the market for the very near contract. We thought that perfect market pricing model would hold in LAST30 at least.

Two step least square procedure could provide the better efficient estimators. The coefficients of the dummy variables in Test I and Test II, therefore, provide more reliable estimations of the dividend risk premium of the S&P 500 for every futures contract to the corresponding one of the NYSE futures contract. We find that S&P 500's dividend risk premium (in absolute value) is larger than that of NYSE's. Recall that our model is constructed in terms of a "physical unit." Therefore, the dividend risk premium is measured by a unit. But one should recall that the transaction value of S&P 500 index futures is currently greater than that of the NYSE index futures by approximately fifty percent. Therefore, it is not surprising that the S&P 500 bears larger dividend risk premium.

Test of the Perfect Market Model

Regressing FD on FCESP using the LAST 30 data set, we expect that both the intercepts and the slopes are not significantly different from zero with low R-square. This is due to the lack of any explanatory power of the FCESP variable. It means that the following model can

test the perfect market model.

Test III: Test of eq(3.12) for LAST30 data set

Step 1: The first step regression

$$\text{Model: } FD(W) = \alpha + \beta \text{ FCESP}(W) + \varepsilon_w \quad \dots\dots\dots(4.8)$$

where

$$1) \quad FD(W) \equiv F(W, T) + \sum_{i=w}^T D(i) \exp(r_f (T-i)/(T-t))$$

$$- I(W) \exp(r_f (T-W)/(T-t))$$

$$2) \quad \text{FCESP}(W) \equiv I(W) \exp(r_f (T-W)/(T-t))$$

$$3) \quad \varepsilon_w = \rho \varepsilon_{w-1} + \xi_w$$

$$4) \quad t < w < T$$

THE NULL HYPOTHESES:

$$(1) \quad \underline{\alpha} = 0$$

$$(2) \quad \beta = 0$$

THE ALTERNATIVE HYPOTHESES:

$$(1) \quad \underline{\alpha} \neq 0$$

$$(2) \quad \beta \neq 0$$

Step 2: The second step regression

$$\text{Model: } \Delta FD(W) = \alpha + \beta \Delta \text{ FCESP}(W) + \xi_w \quad \dots\dots\dots(4.9)$$

where

$$1) \quad \Delta FD(W) = FD(W) - \widehat{\rho} FD(W-1)$$

$$2) \quad \Delta \text{FCESP}(W) = \text{FCESP}(W) - \widehat{\rho} \text{FCESP}(W-1)$$

$$3) \quad t < w < T$$

THE NULL HYPOTHESES:

$$(1) \quad \underline{\alpha} = 0$$

$$(2) \quad \beta = 0$$

THE ALTERNATIVE HYPOTHESES:

$$(1) \quad \underline{\alpha} \neq 0$$

$$(2) \quad \beta \neq 0$$

The results are shown in Table 4.8. None of the parameters are significant. The perfect market pricing model is confirmed in the LAST30 data set. This is true for all contracts, including the very earlier stage contracts, such as March 1983 contract. This suggests that the index futures market was in equilibrium at an earlier stage than other researchers have suggested.

Supportive Tests: Like-CAPM and the Hedge Relation

Dusak (1973) starts from the CAPM to explain the "return" of a futures contract in order to study individual commodity futures. The following equation is employed.

$$\frac{E(\tilde{P}(i,1)) - P'(i,0)}{p(i,0)} = B_i (E(\tilde{R}_F) - r_f)$$

where

$\tilde{R}_F$: the return of the market portfolio

r_f : risk-free rate

$P(i,0)$: spot price of i th commodity at period 0

$\tilde{P}(i,1)$: spot price of i th commodity at period 1

$P'(i,0)$: futures price of i th commodity at period 0

The above form is similar to our hedging relation. However, the model was obtained from a heuristic discussion as opposed to an equilibrium derivation which was used in this research.

The test hypotheses and the regression used to test the Like-CAPM and the hedging relation are follows:

Test IV: Test of eq(3.16), the Like-CAPM

$$\text{Model: } \tilde{r}_F = \alpha + \beta (\tilde{r}_S - r_f) \quad \dots\dots\dots(4.10)$$

THE NULL HYPOTHESES:

$$H_0: \alpha = 0, \quad \beta = 1$$

THE ALTERNATIVE HYPOTHESIS:

$$H_1: \alpha \neq 0 \quad \text{or} \quad \beta \neq 1$$

Test V: Test of eq(3.17), the hedging relation

$$\text{Model: } \widetilde{R}_F = \alpha + \beta(\widetilde{r}_S - r_f)$$

THE NULL HYPOTHESIS:

$$H_0: \alpha = 0, \quad \beta = 1$$

THE ALTERNATIVE HYPOTHESIS:

$$H_1: \alpha \neq 0 \quad \text{or} \quad \beta \neq 1$$

Both regressions' null hypotheses state that the intercepts are zero and the slopes are one. Under the dividend risk argument, we also expect the intercepts of eq(4.7) and eq(4.8) to be zero. But the dividend risk argument suggests that the slope of eq(4.7) and eq(4.8) will be significant and greater than one. As noted before, serial correlation would be adjusted if necessary. However, performing ordinary least square at the rate of return level might adequately avoid serial correlation in general.

The above two tests are performed on all of the seven data subsets. The results of the Like-CAPM test are reported in Tables 4.9-4.12. No intercepts of the regressions are significantly different from zero. The statistical insignificance of the intercept term only indicates that the regression line does go through the origin. This suggests that the margin requirement does not affect the index futures pricing and that no abnormal return is earned corresponding to its systematic risk. The index futures is priced efficiently in general, even in the early stage of the index futures market development. Examining the trend of the slope coefficient over the different data subsets, we find some evidence that

earlier observations have greater slope values. This is consistent with the belief that less dividend risk is priced in the market as time approaches the maturity date of the index futures contract. This result is consistent with Test I suggesting that the effect of dividend risk decrease over time to the maturity. The Like-CAPM test actually is derived from Test I, after we have defined the term "return" of index futures contract. It is therefore not surprising that the results are not changed. In Table 4.12, the slope is not significantly different from one. This would support our conclusion that the perfect market pricing model holds in LAST30.

The results of the hedge relationship are reported in Table 4.13-4.16. Again, the intercepts are not statistically significant over most contracts. However, the main idea of this test is to examine the behavior of hedging overnight. Thus, our main concern is the slope coefficient. The results, however, do not lend support to the claim of the null hypothesis that the slopes, in general, are equal to one. It means that the overnight hedge ratio is less than one since the slope is greater than one. One day perfect hedge of one index futures to one index-equivalent portfolio is impossible. Dividend risk, along with holding period and interest rate change, appear to be important factors for short term hedgers to consider.

In sum, the extended analysis suggests that the basic findings is robust for many different definitions of the variables used in the analysis. The results are basically supportive of the dividend risk hypothesis since they adequately explain the futures discount, in the sense that the dividend risk premium is priced in the market and is negative for the investor who shorts the index futures.

However, there is one puzzling aspect of the results in most tables. The slope coefficients of the FCESP are significantly different from zero. This variation in the reported value may be caused by four reasons. In the first place, it might be due to a term structure problem; thus a multiperiod pricing model should be developed to guide the further empirical study. In the second place, it might be due to a measurement error of the interest rate series for its unmatched maturity date to futures price series. In the third place, the approximation technique is not totally accurate regarding the collected dividend payout series. Finally, there might exist the liquidity risk associated with the maturity dynamics of the earlier days' trading. Future research should be done in this direction.

Summary

This chapter presents an empirical test of whether a dividend risk is priced in the index futures market and whether the pricing model explains the structure under study. We empirically investigate the closed forms developed in Chapter III. The results support the dividend risk argument. Futures discounts could be explained by a factor which increases with seasonality as the time to maturity increases. Dividend uncertainty is resolved by dividend announcements and as the contract approaches maturity. The unit dividend risk premium is estimated using the pricing model developed in this dissertation. Due to the greater transaction value of S&P 500 index futures, S&P 500's unit dividend risk premium is priced more in the market than NYSE index futures'. Of course, one has to interpret these comparison with considerable caution. In sum, the results reported in this chapter may be

of use in analyzing historical price change, evaluating market efficiency, and developing strategies to identify and exploit arbitrage opportunities.

However, the use of models of equilibrium pricing under uncertainty in empirical study of futures markets is very limited. As yet, no precise pricing model of index futures contracts has been found which offers completely satisfactory explanatory power, though some progress has been made here.

Notes

- [1] Cox, Ingersoll, and Ross (1978) examine the theoretical differences between forward and futures prices in a variety of contexts. However, Cornell and Reinganum (1981), and Elton, Gruber, and Rentzler (1982) indicate that the difference is economically insignificant.
- [2] The reason to include FCESP in the right hand side is to make the test completely meet the econometric considerations. See Pindyck and Rubinfeld (1981), p. 82.
- [3] However, if we include the intercept in these regression a new multicollinearity arises.

TABLE 4.1

Frequency of Days Between the Announcement Date and the Ex-date of Dividend Events in NYSE and S & P 500 1982-1983*

Days	Accumulated Percentage of NYSE	Accumulated Percentage of S & P 500
5	0.06	0.06
10	0.34	0.36
15	0.51	0.53
20	0.60	0.61
25	0.68	0.68
30	0.75	0.75
35	0.81	0.81
40	0.85	0.86
45	0.90	0.90
50	0.93	0.93
55	0.95	0.95
60	0.97	0.97
98	1.00	1.00

Note: * Dividend Code of CRSP tape, 1212, 1232, 1239, 3225, 3285, 3723, 2763, 3823, 3825, 3863, 4533, 4822, 5523, 5533, 6521, are excluded. In addition, 20 over the counter companies of S & P 500 are not available. In sum, NYSE has 11992 dividend events and S & P 500 has 4550 dividend events

TABLE 4.2

1981-83 Monthly Dividend Yields for the NYSE Composite Index#

<u>Month</u>	<u>1983</u>	<u>1982</u>	<u>1981</u>
January	3.49%	2.16%	2.24%
February	13.68%	9.84%	8.39%
March	6.69%	4.92%	3.83%
April	4.62%	3.60%	2.31%
May	14.57%	10.92%	8.15%
June	6.38%	4.08%	3.87%
July	3.46%	2.76%	2.24%
August	13.99%	11.76%	8.90%
September	7.70%	4.08%	3.72%
October	4.65%	3.36%	4.15%
November	14.27%	9.24%	8.68%
December	6.50%	3.00%	3.63%

Note:

- # The dividend yields are measured by the difference between the monthly value-weighted return, including dividends, and the value-weighted return, excluding dividends, for the New York Stock Exchange Stocks. The yields have been converted to annual estimates by multiplying the monthly estimates by 12. The data is from CRSP monthly tape.

Test I: $FD = \alpha M + \beta FCESP$
Second Step Least Square

Data Set: WHOLE
Market Index:

S & P 500

NYSE

Contract # (Maturating)	8206	8209	8212	8303	8306	8309	8312	8312	8403
M1	na	na	na	-0.79*	-1.08**	-0.89**	-1.43**	-0.57**	-0.47**
M2	na	na	na	0.71	0.89**	0.90**	0.84**	0.46**	0.46**
M3	na	na	na	-7.09	0.05	0.44	-0.04	-0.01	-0.06
M4	0.08	0.53	1.32**	0.38	-1.43**	-1.31**	-1.36	-0.53**	-0.48**
M5	-0.41	-0.48	-0.38	-0.08	-0.13	-0.65*	-0.58**	-0.37**	-0.27**
M6	-2.99	-1.53**	-2.33**	-2.49**	-11.63**	-1.11**	-0.91**	-0.40**	-0.47**
M7	na	0.81**	1.10**	1.35**	0.53**	-1.19**	-0.91**	-0.31*	-0.15
M8	na	0.09	0.61	0.58	0.70	-16.45**	-0.93**	-0.48**	-0.47**
M9	na	-4.87	-0.75	-0.43	-0.29	0.54**	-0.64**	-0.45**	-0.37**
M10	na	na	-0.61	-0.73*	-0.28	-1.45	-0.71**	-0.36**	-0.39**
M11	na	na	0.05	-0.28	-0.75**	na	0.02	-0.06	-0.18
M12	na	na	-7.01*	-0.82*	-0.81**	-0.89	-3.53	-2.93	-1.28
FCESP	0.07	0.08	0.07	0.06	0.07**	0.11**	0.03	0.05	0.02
R ²	0.29	0.61	0.86	0.91	0.96	0.97	0.98	0.97	0.98
DW	1.69	1.50	1.85	1.99	1.87	2.06	1.90	2.04	2.27
ρ	0.10	0.24	0.07	0.00	0.03	0.02	0.01	0.01	-0.04
df	28	73	118	143	121	114	156	171	171

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

8206 stands for YMM, meaning that June 1982 contract.

Others are the same format.

Test 1: $FD = \alpha M + \beta FCSP$
Second Step Least Square

Data Set: IE30
Market Index:

S & P 500

NYSE

Contract (Maturing)	8206	8209	8212	8303	8306	8309	8312	8312	8403
M1	na	na	na	-0.67	-1.09**	-0.89**	1.43**	-0.60**	-0.54**
M2	na	na	na	3.57Δ	0.89**	-0.92**	0.85**	0.48**	-0.48**
M3	na	na	na	-10.28Δ	0.04	0.46	-0.03	0.01	-0.05
M4	0.29	0.38	1.31**	0.37	-1.42**	-1.29**	-1.36**	-0.57**	-0.55**
M5	-3.97Δ	-0.84	-0.40	-0.08	7.70Δ	-0.65*	-0.58**	-0.39**	-0.31*
M6	10.21Δ	-1.43*	-2.32**	-2.50**	-19.05Δ	-1.12**	-0.91**	-0.43**	-0.50*
M7	na	0.36	1.09**	1.35	0.53	-1.16**	-0.92**	-0.34**	-0.18
M8	na	39.18Δ	0.64	0.59	0.66	-17.24	-0.92**	-0.56**	-0.52**
M9	na	-48.72Δ	-0.72	-0.43	-0.32	0.60	-0.64**	-0.49**	-0.43**
M10	na	na	-0.69	-0.73	-0.30	-1.45	-0.84**	0.41*	-0.43**
M11	na	na	10.47Δ	-0.28	-0.75*	na	-0.20Δ	0.76Δ	-0.20
M12	na	na	-17.67Δ	-0.82*	-0.91**	-0.88	-3.54Δ	-3.89Δ	-1.06
FCSP	0.26	0.18	0.07	0.06	0.07*	0.11**	0.03	0.05	0.01
R ²	0.85	0.62	0.87	0.91	0.96	0.97	0.98	0.98	0.98
DW	1.74	1.42	1.82	2.00	1.86	2.06	1.90	1.93	2.08
p	0.01	0.25	0.08	0.01	0.02	0.02	0.02	0.03	0.01
df	10	56	102	126	110	104	140	1.62	171

Note: * Significant at the 5% level of significance
 ** Significant at the 1% level of significance
 Δ Perfect multicollinearity

TABLE 4.5

Test IV: $FD = \alpha \quad \underline{S} + \beta \quad FCESP$

Second Step Least Square

Data Set: WHOLE

Contract	2 month before	1 month before	during	FCESP	R ²	DW	ρ	df
S & P 500:								
8206	-2.9925	-0.0801	-0.4134	0.4134	0.29	1.69	0.10	28
8209	-1.4823**	0.7420**	-0.2742	0.0216**	0.60	1.47	0.25	76
8212	-1.2417**	0.3397	0.0699	0.0113**	0.72	1.98	0.00	124
8303	-0.8653**	-0.0594	0.1075	0.0072**	0.75	2.19	-0.10	152
8306	-0.6663**	-0.8150**	0.0479	0.0016	0.91	1.87	0.03	130
8309	-0.4722**	-0.7632**	-0.0782	0.0039	0.89	2.28	-0.03	122
8312	-0.3879**	-0.6263**	-0.0942	0.0087**	0.91	2.13	-0.09	165
NSYE:								
8312	-0.2124**	-0.2077**	-0.0540	0.0063**	0.88	2.30	-0.11	180
8402	-0.2205**	-0.0738	-0.0511	0.0182**	0.81	2.49	-0.16	180

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

TABLE 4.6

Test IV: $FD = \alpha \underline{S} + \beta \text{ FCESP}$

Second Step Least Square

Data Set: LE30

Contract	2 month before	1 month before	during	FCESP	R ²	DW	ρ	df
S & P 500:								
8209	-1.3500**	0.6906*	-0.3333	0.0231	0.60	1.4140	-0.2611	59
8212	-0.3498**	0.4474*	0.1540	0.0082	0.72	0.9297	0.0345	108
8303	-0.9157*	-0.0395	0.0755	0.0056	0.78	2.1905	-0.0963	135
8306	-0.6659**	-0.8098**	0.0433	0.0015	0.92	0.8701	0.0372	119
8309	0.4696*	-0.7531**	-0.0758	0.0039	0.89	2.2883	-0.0291	112
8312	-0.3569**	-0.6730**	-0.1370	0.0099	0.92	2.1349	-0.0927	149
NYSE:								
8312	-0.2293**	-0.2387**	-0.0674	0.0065*	0.90	2.0932	-0.0541	171
8403	-0.2389**	-0.1097	-0.0687	0.0166**	0.87	2.2406	-0.0929	180

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

Test IV: $FD = \alpha \frac{S}{S} + \beta FCESP$
Second Step Least Square

	2 Month before	1 Month before	during	FCESP	R ²	DW	ρ	df	Data Set
S&P 500									
8212	-1.8667**	-1.1869**	0.0429	0.0112**	0.75	1.84	0.02	43	LAST90
	-1.5864**	-1.6057**	0.2042	0.0112**	0.86	2.00	-0.00	43	L120LE30
	-1.8085**	0.9664**	-0.0422	0.0160*	0.86	1.48	0.25	76	LE90
	-1.6662**	0.9697**	-0.1675	0.0174	0.85	1.44	0.27	60	LE120
8303	-0.6858*	-1.0411**	0.6211*	0.0017	0.68	1.95	0.00	43	LAST90
	-1.0860	-0.7261	-0.6531	0.0144	0.83	1.98	-0.00	41	L120LE30
	-1.0413**	0.1913	0.1612	-0.0004	0.82	2.18	-0.09	104	LE90
	-1.1745**	0.2431	0.3182	-0.0044	0.83	2.12	-0.07	89	LE120
8306	-0.0465	-1.4533	0.0223	-0.0036	0.86	1.78	0.11	39	LAST 90
	-0.3237	-1.3649	1.0094	-0.0098	0.89	1.78	0.10	43	L120LE30
	-1.2497**	-0.6957**	0.0401	0.0076**	0.95	1.79	0.04	87	LE90
	-1.1532**	-0.6213*	-0.4020	0.0079	0.96	1.78	0.02	71	LE120
8309	-0.2531	-0.3086	8.8376	-0.0695	0.85	2.02	-0.03	36	LAST90
	-0.6485**	-0.3852	-0.2731	0.0067	0.76	1.92	0.04	42	L120LE30
	-0.2636	-1.5329	0.2014	-0.0057	0.96	1.96	0.06	81	LE90
	-0.3397	-1.7241	0.4360	-0.0078	0.97	1.98	0.04	65	LE120
8312	-0.3343*	-0.5339**	0.0958	0.0029	0.59	1.94	0.01	43	LAST90
	-0.4086**	-0.5969**	-0.6680**	0.0128**	0.85	1.92	0.01	43	L120LE30
	-0.3549*	-0.8384**	-0.0441	0.0048	0.94	2.05	0.06	117	LE90
	-0.4295	-0.8290	0.0781	0.0028	0.94	2.08	-0.08	101	LE120
NYSE									
8312	8.5983**	-0.1328	0.0531	-0.1361**	0.74	1.88	0.09	35	LAST90
	-0.2972**	-0.1952	4.4184	-0.0871	0.82	1.60	0.13	42	L120LE30
	-0.1747**	-0.2953**	-0.1034	0.0016	0.93	2.05	-0.03	141	LE90
	-0.2468*	-0.3247**	0.0064	-0.0025	0.94	2.07	-0.04	125	LE120

Note: * Significant at the 5% level of significance.

** Significant at the 1% level of significance.

Data Set: LAST 30

Regression Type: First Stage Least Square Second Stage Least Square

coefficient α β R^2 DW ρ df α β R^2 DW ρ

S&P 500

8209#	-4.34	0.03	0.05	1.65	0.16	14	-3.13	0.03	0.02	1.68	0.14
8212#	-7.16	0.05	0.05	1.84	0.06	13	-5.76	0.04	0.03	1.98	-0.00
8303	28.325*	-0.19	0.35	1.33	0.27	14	18.60	-0.17	0.21	1.75	0.09
8306##	-25.99**	0.16	0.41	1.55	0.19	13	-19.97*	0.15*	0.32	1.90	0.03
8309	-6.36	0.04	0.03	1.38	0.20	14	-4.38	0.03	0.04	1.74	0.02
8312	1.64	-0.01	0.00	1.69	0.06	13	1.36	-0.01	0.00	1.78	0.02

NYSE

8312	6.18	-0.08	0.15	1.52	0.04	6	6.21	-0.07	0.16	1.54	0.03
------	------	-------	------	------	------	---	------	-------	------	------	------

Note: * Significant at 5% significance level

** Significant at 1% significance level

Joint tests of null hypotheses, significant at 5% significance level

Joint tests of null hypotheses, significant at 1% significance level

TABLE 4.9

$$\text{Test II: } \bar{r}_F = \alpha + \beta_F (\bar{r}_S - r_f)$$

Data set: WHOLE							
<u>Market</u>	<u>Contract</u>	<u>α</u>	<u>β_F</u>	<u>R^2</u>	<u>DW</u>	<u>ρ</u>	<u>df</u>
S & P 500							
	8209	-0.0002	1.21**	0.78	2.28	-0.16	78
	8212	-0.0004	1.20**	0.78	2.58	-0.29	126
	8303	-0.0003	1.20**	0.75	2.83	-0.42	154
	8306	-0.0004	1.02	0.70	2.57	-0.27	132
	8309	-0.0005	1.20**	0.74	2.79	-0.33	124
	8312	-0.0000	1.08	0.79	2.70	-0.37	167
NYSE							
	8308	-0.0008	1.29**	0.80	2.52	-0.24	41
	8306	0.0002	1.57	0.20	2.57	-0.28	81
	8309	-0.0001	1.02	0.54	3.08	-3.08	127
	8312	-0.0004	1.16**	0.80	2.51	-0.23	182
	8403	-0.0004	1.16**	0.77	2.67	-0.23	182

Note: * Significant at the 5% level of significance
 ** Significant at the 1% level of significance

TABLE 4.10

$$\text{Test II: } \tilde{r}_F = \alpha + \beta_F (\tilde{r}_S - r_f)$$

Data set: LE30							
<u>Market</u>	<u>Contract</u>	<u>α</u>	<u>β_F</u>	<u>R^2</u>	<u>DW</u>	<u>ρ</u>	<u>df</u>
S & P 500							
	8209	0.0005	1.36**	0.72	2.19	-0.13	61
	8212	-0.0006	1.23**	0.78	2.56	-0.28	110
	8303	-0.0002	1.21**	0.75	2.87	-0.44	137
	8306	-0.0004	1.01**	0.70	2.57	-0.27	121
	8309	-0.0007	1.23**	0.74	2.83	-0.35	114
	8312	0.0000	1.08**	0.78	2.70	-0.37	151
NYSE							
	8303	-0.0008	1.33**	0.83	2.79	-0.41	28
	8306	0.0002	1.59	0.20	2.56	-0.28	77
	8309	-0.0001	1.02	0.54	3.05	-0.53	126
	8312	-0.0004	1.16**	0.77	2.45	-0.23	173
	8403	-0.0004	1.16**	0.77	2.57	-0.23	182

Note: * Significance at the 5% level of significance
 ** Significance at the 1% level of significance

TABLE 4.11

$$\text{Test II: } \tilde{r}_F = \alpha + \beta_F (\tilde{r}_S - r_f)$$

Contract S & P 500:	α	β_F	R^2	DW	ρ	df	Data Set
8209	0.0003	1.23*	0.81	2.41	0.25	45	LAST90
	0.0014	1.54**	0.74	2.46	-0.24	45	L120LE30
	-0.0002	1.16	0.69	1.74	-0.05	30	LE90
	-0.0009	0.88	0.84	2.37	-0.30	13	LE120
8212	-0.0004	1.18	0.77	2.98	-0.49	45	LAST90
	-0.0015	1.24*	0.78	2.87	-0.44	45	L120LE30
	-0.0005	1.23	0.79	2.28	-0.14	78	LE90
	-0.0000	1.26**	0.78	2.18	-0.11	62	LE120
8303	-0.0010	1.27**	0.84	2.48	-0.27	45	LAST90
	-0.0005	1.24**	0.84	2.75	-0.40	43	L120LE30
	-0.0003	1.20**	0.73	2.89	-0.44	106	LE90
	-0.0001	1.23**	0.73	2.85	-0.44	91	LE120
8306	-0.0009	1.23*	0.84	2.32	-0.20	41	LAST90
	-0.0012	1.23*	0.82	2.14	-0.07	45	L120LE30
	-0.0002	0.98	0.68	2.58	-0.27	89	LE90
	-0.0001	0.97	0.67	2.63	-0.29	73	LE120
8309	0.0002	0.96	0.88	2.94	-0.47	38	LAST90
	0.0002	1.03	0.84	2.64	-0.33	44	L120LE30
	-0.0013	1.38**	0.74	2.93	-0.36	83	LE90
	-0.0015	1.40**	0.73	3.05	-0.38	67	LE120
8312	-0.0004	0.94	0.86	2.49	-0.25	45	LAST90
	0.0002	0.93	0.88	2.52	-0.27	45	L120LE30
	-0.0001	1.12*	0.77	2.73	-0.39	119	LE90
	-0.0002	1.15	0.76	1.17	-0.00	103	LE120
NYSE							
8312	-0.0004	1.03	0.85	2.19	-0.07	37	LAST90
	0.0001	0.99	0.85	2.11	-0.09	44	L120LE30
	-0.0005	1.19**	0.79	2.49	-0.25	143	LE90
	-0.0007	1.21	0.80	2.49	-0.25	127	LE120

Note: * Significant at the 5% level of significance
 ** Significant at the 1% level of significance

TABLE 4.12

$$\text{Test II: } \tilde{r}_F = \alpha + \beta_F (\tilde{r}_S - r_f)$$

Data Set:	LAST30						
<u>Market</u>	<u>Contract</u>	α	β_F	R^2	<u>DW</u>	<u>ρ</u>	<u>df</u>
S & P 500:							
	8206	0.0016	1.29	0.73	2.51	-0.26	15
	8209	-0.0018	1.24	0.84	2.44	-0.30	14
	8212	-0.0002	1.08	0.82	2.51	-0.30	13
	8303	-0.0003	1.05	0.81	2.10	-0.08	14
	8306	-0.0000	1.08	0.85	2.42	-0.27	13
	8309	0.0004	0.92	0.86	2.48	-0.28	14
	8212	-0.0010	1.05	0.79	2.50	-0.30	13
NYSE							
	8312	-0.0009	1.01	0.75	1.53	-0.05	6

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

TABLE 4.13

Test III: $\tilde{R}_F = \alpha + \beta_H (\tilde{r}_S - r_f)$

Data Set: WHOLE							
<u>Market</u>	<u>Contract</u>	<u>α</u>	<u>β_H</u>	<u>R^2</u>	<u>DW</u>	<u>ρ</u>	<u>df</u>
S & P 500:							
	8209	-0.0002	1.22**	0.78	2.29	-0.16	78
	8212	-0.0004	1.20**	0.78	2.59	-0.30	126
	8303	-0.0003	1.21**	0.75	2.84	-0.42	154
	8306	-0.0004	1.02	0.70	2.57	-0.27	132
	8309	-0.0006	1.22	0.74	2.78	-0.33	124
	8312	-0.0000	1.10*	0.78	2.69	-0.37	167
NYSE							
	8303	-0.0008	1.30**	0.80	2.51	-0.24	41
	8306	-0.0002	1.60	0.20	2.64	-0.32	81
	8309	-0.0001	1.04	0.53	3.09	-0.54	127
	8312	-0.0004	1.18**	0.80	2.50	-0.23	182
	8403	-0.0004	1.18**	0.77	2.67	-0.24	182

Note: * Significance at the 5% level of significance

** Significance at the 1% level of significance

TABLE 4.14

Test III: $\tilde{R}_F = \alpha + \beta_H (\tilde{r}_S - r_f)$

Data Set: LE30							
<u>Market</u>	<u>Contract</u>	<u>α</u>	<u>β_H</u>	<u>R^2</u>	<u>DW</u>	<u>ρ</u>	<u>df</u>
S & P 500							
	8209	0.0005	1.36**	0.72	2.20	-0.14	61
	8212	-0.0006	1.24**	0.78	2.57	-0.29	110
	8303	-0.0003	1.22**	0.75	2.88	-0.44	137
	8306	-0.0004	1.02**	0.70	2.57	-0.27	121
	8309	-0.0007	1.25**	0.74	2.82	-0.35	114
	8312	-0.0000	1.10*	0.78	2.69	-0.37	151
NYSE							
	8303	-0.0009	1.34**	0.83	2.79	-0.41	28
	8306	-0.0002	1.62	0.20	2.64	-0.32	77
	8309	-0.0001	1.04	0.54	3.05	-0.52	126
	8312	-0.0004	1.18**	0.80	2.45	-0.23	173
	8403	-.00004	1.19**	0.77	2.58	-0.23	182

Note: * Significance at the 5% level of significance

** Significance at the 1% level of significance

TABLE 4.15

Test III: $\tilde{R}_F = \alpha + \beta_H (\tilde{r}_S - r_f)$

Contract	α	β_H	R^2	DW	ρ	df	Data Set
S & P 500							
8209	-0.0003	1.23*	0.81	2.41	0.25	45	LAST90
	0.0014	1.54**	0.74	2.47	-0.24	45	L120LE30
	-0.0002	1.17	0.70	1.76	-0.04	30	LE90
	-0.0010	0.90	0.85	2.37	-0.30	13	LE120
8212	-0.0004	1.18	0.77	2.99	-0.50	45	LAST90
	-0.0016	1.24*	0.78	2.88	-0.44	45	L120LE30
	-0.0005	1.24**	0.79	2.29	-0.15	78	LE90
	-0.0000	1.27**	0.78	2.18	-0.11	62	LE120
8303	-0.0010	1.27**	0.84	2.48	-0.27	45	LAST90
	-0.0006	1.25**	0.83	2.76	-0.40	43	L120LE30
	-0.0001	1.20**	0.73	2.90	-0.45	106	LE90
	-0.0001	1.23**	0.73	2.85	-0.44	91	LE120
8306	-0.0009	1.23*	0.83	2.33	-0.20	41	LAST90
	-0.0012	1.23*	0.82	2.14	-0.07	45	L120LE30
	-0.0003	1.00	0.68	2.58	-0.27	89	LE90
	-0.0002	0.99	0.67	2.63	-0.29	73	LE120
8309	0.0002	0.97	0.88	2.93	-0.47	38	LAST90
	0.0002	1.02	0.84	2.64	-0.32	44	L120LE30
	-0.0014	1.40**	0.74	2.93	-0.36	83	LE90
	-0.0016	1.42	0.73	3.05	-0.39	67	LE120
8312	-0.0004	0.94	0.87	2.49	-0.25	45	LAST90
	0.0002	0.95	0.88	2.53	-0.27	45	L120LE30
	-0.0005	1.14*	0.77	2.73	-0.38	119	LE90
	-0.0002	1.17*	0.76	2.27	-0.41	103	LE120
NYSE							
8312	-0.0004	1.04	0.85	2.19	-0.07	37	LAST90
	0.0001	1.00	0.85	2.11	-0.10	44	L120LE30
	-0.0005	1.21**	0.79	2.49	-0.25	143	LE90
	-0.0007	1.23	0.79	2.49	-0.25	127	LE120

Note: * Significance at the 5% level of significance
 ** Significance at the 1% level of significance

TABLE 4.16

Test III: $\tilde{R}_F = \alpha + \beta_H (\tilde{r}_S - r_f)$

Data Set: LAST30

<u>Market</u>	<u>Contract</u>	<u>α</u>	<u>β_H</u>	<u>R^2</u>	<u>DW</u>	<u>ρ</u>	<u>df</u>
S & P 500:							
	8206	0.0015	1.28	0.73	2.51	-0.26	15
	8209	-0.0018	1.24	0.85	2.43	-0.30	14
	8212	-0.0002	1.07	0.82	2.52	-0.30	13
	8303	-0.0003	1.05	0.81	2.08	-0.07	14
	8306	-0.0000	1.08	0.85	2.42	-0.27	13
	8309	0.0004	0.93	0.86	2.48	-0.28	14
	8312	-0.0010	1.05	0.79	2.50	-0.30	13
NYSE							
	8312	-0.0009	1.01	0.75	1.53	-0.04	6

Note: * Significant at the 5% level of significance
 ** Significant at the 1% level of significance

CHAPTER V CONCLUSION

The adjustment for dividend uncertainty, either constant or stochastic, appears to reduce index futures prices. The extent of the difference between the constant and stochastic cases depends on the size of the constant dividend yield, the size of the variance of the dividend process, and the extent correlation between the return on the index-equivalent portfolio and the dividend yield. As time to maturity increases, the difference is magnified.

Using a market equilibrium approach, a model which includes a dividend risk premium was developed. This dividend risk premium can not be hedged away, if we buy a classic risk-arbitrage portfolio which goes long one unit of the index-equivalent portfolio and short one unit of the index futures simultaneously. In addition, the risk level of the futures contract relative to the index-equivalent portfolio has been examined.

In Chapter III, three empirically testable equations were derived. The first specifies the price relationship between the spot and the index futures which includes the effect of a random dividend. The second is a Like-CAPM model which shows the corresponding systematic risk level of index futures contracts. Finally, a hedge relationship was examined. Based on these three equations, five tests were performed on seven data sets to examine the dividend risk behavior empirically.

In Chapter IV, the dividend risk argument is examined empirically. The empirical results show that the dividend risk premium with

seasonality the closer the contract is to maturity. The total risk premium two months before the dividend-peak-month is large; one month before the dividend-peak-month it is somewhat smaller, but still significant; and during the dividend-peak-month it is not significantly different from zero. As to the magnitude, S&P 500 has larger market value with a larger associated dividend risk premium in general. In addition, the nearest contract appears to be priced according to the perfect market pricing model, which implies that the dividend risk is not important for the last thirty trading-days at least.

Since we have examined the daily price of index futures in New York Futures Exchange and Chicago Board Futures Exchange over a one and a half year time period, inferences drawn from this research must be tentative. However, we have analyzed in excess of four contracts in each market and have found similarities in the results for index futures on these two markets. Therefore, we consider this research to be one of the most extensive empirical examinations of index futures price to be reported in the literature to date.

APPENDIX A
DATA BASES

TABLE A.1
Data Base of Index Futures Data

<u>Market Index</u>	<u>Contract</u>	<u>Start-Trading Date</u>	<u>End-Trading Date</u>	<u>Number of Observations</u>
NYSE***				
	8206*	820506**	820629	38
	8209	820506	820929	102
	8212	820506	821230	166
	8203	820506	830330	229
	8206	820630	830629	253
	8306	820722	830929	302
	8312	820723	831229	364
	8403	820930	840329	379
S&P 500****				
	8206	820421	820617	41
	8209	820421	820916	104
	8212	820421	821216	168
	8303	820421	830317	213
	8306	820625	830616	202
	8309	820920	830915	196
	8312	821223	831215	232
	8403	830325	840308	200

Note: * 8206 stands for YYMM, meaning index futures contract maturity on June 1982. Others are the same format
 ** 820506 stands for YYMMDD, meaning the date of May 6, 1982. Others are the same format
 *** The data source of NYSE is directly provided by statistics department NYSE. Data are in 4:15 P.M. New York time.
 **** The data source of S&P 500 is from a ticker by ticker tape, provided by Chicago Board. The close price at 4:15 P.M. is sorted as the data base.

TABLE A.2

Data other than Index Futures ****

<u>Market</u>	<u>Data type</u>	<u>Start-Date</u>	<u>End-Date</u>
NYSE	Spot	820104	840329
	Dividend*	830101	831231
S & P 500	Spot	820421	840308
	Dividend**	820104	831231
	Treasury-Bill***	820421	840308

- Note:
- * Dividend payout of market index is provided by statistics department, NYSE.
 - ** Dividend payout of S & P 500 is collected from CRSP daily tape, using December 1982 name list in COMPUSTAT tape. This data series is a approximation value, 6.6% number of companies series is a approximation value, 6.6% number of companies missing or 3.6% dividend payout of total dividends missing, based on the value of December 31, 1982. COMPUSTAT tape.
 - *** Treasury-Bill Futures data is sorted from a ticker by ticker tape, provided by Chicago Board.
 - **** Market value of S & P 500 and NYSE are computed from CRSP tape. The market value of S & P 500 underestimates 3.6% value of total value, based on the value of December 31, 1982 COMPUSTAT tape. The names of missing companies are available upon request.

APPENDIX B
RESULT OF FIRST STEP
LEAST SQUARE

TABLE B.1

Test I: $FD = \alpha M + \beta$ FCESP
First Step Least Square

Data Set: WHOLE		S&P 500							NYSE		
Market Index											
Contract (Maturity)	8206	8209	8212	8303	8306	8309	8312	8312	8312	8403	
M1	na	na	na	-0.99**	-1.16**	-1.07**	-1.79**	-0.85**	-0.85**	-1.00**	
M2	na	na	na	0.85**	0.97**	0.99**	0.93**	0.52**	0.52**	0.59**	
M3	na	na	na	-8.81*	0.11	0.52	0.08	0.05	0.05	0.08	
M4	0.47	1.28**	1.88**	0.48	-1.53**	-1.49**	-1.85**	-0.92**	-0.92**	-1.04**	
M5	-0.31	-0.45	-0.40	0.05	-0.18	-0.75**	-0.79**	-0.60**	-0.60**	-0.62**	
M6	-2.10	-3.09**	-3.34**	-3.50**	-13.73**	-1.25**	-1.20**	-0.64**	-0.64**	-0.76**	
M7	na	1.44**	1.48**	1.79**	0.56	-1.36**	-1.21**	-0.49**	-0.49**	-0.52**	
M8	na	-0.30	0.81	0.67	0.86	-18.65**	-1.20**	-0.72**	-0.72**	0.80**	
M9	na	-4.99	-0.92*	-0.55	-0.16	0.84	-0.93**	-0.78**	-0.78**	-0.85**	
M10	na	na	-0.88*	-1.01**	-0.30	-1.61	-0.93**	-0.52**	-0.52**	-0.73**	
M11	na	na	0.39	-0.42	-0.85**	na	-0.02	-0.12	-0.12	-0.38**	
M12	na	na	-9.99**	-1.15**	-0.96**	-1.01*	-4.42	2.67	2.67	-0.41	
FCESP	0.01	0.03	0.07**	0.05***	0.008**	0.10**	0.03**	0.02	0.02	-0.00**	
R ²	0.69	0.78	0.90	0.94	0.97	0.97	0.99	0.98	0.98	0.99	
DW	0.67	0.98	1.4	1.48	1.76	1.83	1.46	1.26	1.26	1.02	
df	0.62	0.50	0.29	0.26	0.08	0.13	0.25	0.38	0.38	0.51	
	28	73	118	143	121	114	156	171	171	171	

Note: * Significant at the 5% level of significance
** Significant at the 1% level of significance

Data Set: LE30		S&P 500					NYSE		
Market Index									
Contract (Maturity)	8206	8209	8212	8303	8306	8309	8312	8312	8403
M1	na	na	na	-0.79	-1.17**	-1.06*	-1.79**	-0.86**	-1.00**
M2	na	na	na	1.37	0.96**	1.01**	0.94**	0.54**	0.60**
M3	na	na	na	-9.47*	0.09	0.54	0.09	0.06	0.09
M4	0.34	1.22*	1.84**	0.48	-1.50**	-1.47**	-1.85**	-0.92**	-1.04**
M5	-3.24	-0.74	-0.37	0.05	7.22	-0.75**	-0.79**	-0.60**	-0.62**
M6	9.58	-3.14**	-3.34**	-3.50**	-20.58	-1.26**	-1.20**	-0.64**	-0.76**
M7	na	1.21	1.49**	1.79**	0.56	-1.33**	-1.23**	-0.50**	-0.52**
M8	na	13.15	0.77	0.67	0.80	-19.42**	-1.20**	-0.72**	0.80**
M9	na	22.41	-0.95*	-0.55	-0.20	0.90	-0.93**	-0.78**	-0.85**
M10	na	na	-1.12*	-1.01**	-0.32	-1.59	-1.09**	-0.55**	-0.73**
M11	na	na	6.78	-0.42	-0.84**	na	-1.40	-0.17	-0.38**
M12	na	na	-15.99	-1.15**	-0.98**	-1.00*	-3.34	3.16	-0.41
FCESP	0.06	0.08	0.06**	0.05**	0.08**	0.11**	0.03**	0.03	-0.00**
R ²	0.88	0.79	0.91	0.94	0.97	0.97	0.99	0.99	0.99
DW	1.59	0.86	1.40	1.49	1.76	1.84	1.46	1.32	1.13
df	0.08	0.55	0.30	0.26	0.08	0.12	0.25	0.34	0.45
	10	56	102	126	110	104	140	171	171

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

TABLE B.3

Test IV: $FD = \alpha + \beta$ FCESP

Data set: WHOLE

Variable	2 month	1 month	during	FCESP	R ²	DW	ρ	df
<u>Contract</u>	<u>before</u>	<u>before</u>						
S&P 500:								
8206	-2.103	0.476	-0.312	0.012	0.69	0.686	0.62	28
8209	-3.052**	-1.399**	-0.399	0.022**	0.78	0.98	0.50	76
8212	-2.439**	0.579*	0.161	0.011**	0.86	0.97	0.51	124
8303	-1.894**	0.124	0.133	0.008**	0.91	0.89	0.55	152
8306	-0.875**	-1.208**	-0.057	0.001	0.95	1.25	0.33	130
8309	-0.681**	-1.492**	-0.272	0.003**	0.96	1.17	0.48	122
8312	-0.860**	-1.597**	-0.444**	0.008**	0.97	0.75	0.60	165

NYSE:

8312	-0.605**	-0.662**	-0.279**	-0.006***	0.96	0.67	0.70	180
8403	-0.606**	-0.724**	-0.388**	0.012***	0.98	0.42	0.81	180

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

TABLE B.4

Test IV: $FD = \alpha + \beta \text{ FCESP}$

Data set: LE30

Variable	2 month	1 month	during	FCESP	R ²	DW	ρ	df
<u>Contract</u>	<u>before</u>	<u>before</u>						
S&P 500:								
8206	9.558	0.342	-3.242	0.060	0.88	1.60	0.08	10
8209	-3.058**	-1.423**	-0.356	0.021**	0.79	0.88	0.54	59
8212	-2.587**	0.704*	0.382	0.008**	0.87	1.01	0.48	108
8303	-1.937**	0.084	0.079	0.006**	0.91	0.94	0.53	135
8306	-0.875**	-1.190**	-0.066	0.001	0.95	1.24	0.33	119
8309	-0.681**	-1.483**	-0.278	1.003**	0.96	1.16	0.49	112
8312	-0.793**	-1.546**	-0.522**	0.009**	0.98	0.76	0.60	149
NYSE:								
8312	-0.606**	-0.664**	-0.277**	-0.006**	0.98	0.82	0.66	171
8403	-0.606**	-0.724**	-0.388**	0.012**	0.98	0.44	0.77	180

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

TABLE B.5

Test IV: $FD = \alpha \underline{S} + \beta \text{FCESP}$

2 month before	1 month before	during	FCESP	R ²	DW	ρ	df	Data Set
S&P 500:								
8212								
-1.780**	-1.114**	0.052	0.011**	0.74	2.002	-0.063	43	LAST90
-1.528**	-1.519**	0.204	0.011**	0.86	2.088	-0.049	43	L120LE30
-3.370**	1.654**	0.041	0.017**	0.92	1.082	0.454	76	LE90
-3.345	1.761	0.068	0.014	0.92	1.011	0.486	60	LE120
8303								
0.701	-1.060	0.629	0.002	0.69	1.912	0.023	43	LAST90
-1.327**	-0.811*	-0.878*	0.015	0.88	1.614	0.184	41	L120LE30
-2.085**	0.302	0.295	0.001	0.93	0.993	0.507	104	LE90
-2.271**	0.362	0.631	-0.003	0.93	1.024	0.487	89	LE120
8306								
0.080	-1959*	-0.052	-0.004	0.91	1.460	0.266	39	LAST90
-0.344	-1994**	1.322*	-0.010	0.94	1.306	0.334	43	L120LE30
-1.472**	-0.848**	-0.000	0.007**	0.96	1.476	0.184	87	LE90
-1.234**	-0673*	-0.448	0.008**	0.97	1.655	0.079	71	LE120
8309								
-0.315	-0.340	12.176	-0.076	0.86	1.534	0.208	36	LAST90
-1.163	-0.727	-0.682	0.008**	0.89	1.064	0.457	42	L120LE30
-0.231	-2.000**	0.205	-0.006**	0.97	1.559	0.225	81	LE90
-0.330	-2.054	0.470	-0.008	0.97	1.698	0.158	65	LE120
8312								
-0.673*	-0.851**	0.033	0.004	0.77	1.180	0.403	43	LAST90
-0.910**	-0.006**	-1.178	0.015**	0.93	1.083	0.439	43	L120LE30
-0.565**	-1.730**	-0.281	0.003	0.98	0.873	0.535	117	LE90
-0.678**	-1.701**	-0.099	0.002	0.98	0.891	0.534	101	LE120
NYSE:								
8312								
15.319**	-0.103	0.030	-0.162**	0.85	1.349	1.349	35	LAST90
-0.692**	-0.237	10.926**	-0.109**	0.93	0.822	0.822	42	L120LE30
-0.355**	-0.743**	-0.326**	0.000	0.98	0.773	0.773	141	LE90
-0.451**	-0.750**	-0.102	-0.003	0.98	0.852	0.852	125	LE120

Note: * Significant at the 5% level of significance

** Significant at the 1% level of significance

LIST OF REFERENCES

- Anderson, R.A., and J. Danthine, "Hedging Diversity in Futures Markets: Backwardation and the Coordination of Plans," Research Paper No. 71A, Columbia School of Business, New York (January 1978).
- Athans, M., "The Importance of Kalman Filtering Methods for Economic Systems," Annals of Economic and Social Measurement 3 (January 1974), 49-64.
- Baron, D.P., "Flexible Exchange Rates Forward Markets, and the Level of Trade," American Economic Review 66 (June 1976), 253-266.
- Black, F., "Equilibrium in the Creation of Investment Goods under Uncertainty," In Studies in the Theory of Capital Markets, edited by Michael C. Jensen. Praeger, New York (1972).
- Black, F., and M. Scholes, "From Theory to a New Financial Product," Journal of Finance 29 (May 1974), 399-412.
- Blume, M., "On the Assessment of Risk," Journal of Finance 26 (March 1976), 1-10.
- Brennan, M.J., "The Supply of Storage," American Economics Review 48 (March 1958).
- Bodie, Z., and Victor I. Rosansky, "Risk and Return in Commodity Futures," Financial Analysis Journal 36 (May-June 1980), 27-39.
- Chow, G., "Tests of Equality between Subsets of Coefficients in Two Linear Regression," Econometrica 28 (July 1960), 591-605.
- Constantinides, G.M., and M.S. Scholes, "Optimal Liquidation of Assets in the Presence of Personal Taxes: Implications for Asset Pricing," Journal of Finance 35 (May 1980), 439-452.
- Cootner, P.H., "Return to Speculations: Telser versus Keynes," Journal of Political Economy 48 (August 1960), 396-404.
- Cootner, P.H., Speculation and Hedging Proceedings of a Symposium on Price Effects of Speculation in Organized Commodity Markets, Food Research Institute Studies, Supplement to Vol. VII, 1967.
- Cornell, B., "Testing the Tax Timing Option Theory: A New Approach," Working Paper #5-84, Graduate School of Management, University of California, Los Angeles (March 1984).
- Cornell, B., and K.R. French, "Taxes and the Pricing of Stock Index Futures," Journal of Finance 38 (June 1983), 663-674.

- Cornell, B., and M.R. Reinganum, "Forward and Futures Prices: Evidence From the Foreign Exchange Markets," Journal of Finance 36 (1981), 1035-1045.
- Cox, J., J. Ingersoll, and S.A. Ross, "A Theory of the Term Structure of Interest Rate." Research Paper No. 468, Graduate School of Business, Stanford University (August 1978).
- Cox, J., J. Ingersoll, and S.A. Ross, "The Relation between Forward Prices and Futures Prices," Journal of Financial Economics 9 (December 1981), 321-346.
- Dumas, B., "The Theory of the Trading Firm Revisited," Journal of Finance 33 (June 1978), 1019-1033.
- Dusak, K., "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums," Journal of Political Economy 81 (December 1973), 1387-1406.
- Dybvig, P.H., "An Explicit Bound on Individual Assets Deviations from APT Pricing in a Finite Economy," Working Paper, School of Management Yale University, New Haven, 1983.
- Ederington, L.H., "The Hedging Performance of the New Futures Markets," Journal of Finance 34 (March 1979), 158-170.
- Elton, E., M. Gruber, and J. Rentzler, "Intra-day Tests of the Efficiency of the Treasury Bill Futures Market." unpublished working paper, New York University, New York 1982.
- Ethier, W., "International Trade and Forward Exchange Market," American Economic Review 63 (June 1973), 68-83.
- Figlewski, S., "Why Are Prices for Stock Index Futures So Low?" Working Paper No. 138, Graduate School of Business, University of California, Berkeley (1983a).
- Figlewski, S., "Hedging with Stock Index Futures: Theory and Application in a New Market," Working Paper No. 139, University of California, (1983b).
- Figlewski, S., "Hedging Performance and Basis Risk in Stock Index Futures," Journal of Finance 39 (July 1984), 657-669.
- Fischer, S., "Call Option Pricing When the Exercise Price is Uncertain, and the Valuation of Index Bond," Journal of Finance 33 (March 1978), 169-186.
- Fisher, L., "Some New Stock-Market Indexes," Journal of Business 196 (January 1966), 191-225.
- French, K.R., "A Comparison of Futures and Forward Price," Journal of Financial Economics 12 (November 1983), 311-342.

- Garbade, K., and Z. Lieber, "On the Independence of Transactions on the New York Stock Exchange," Journal of Banking and Finance 1 (September 1977), 151-172.
- Garbade, K., and W. Silber, "Dominant and Satellite Markets: A Study of Dually-Traded Securities," Review of Economics and Statistics 61 (August 1979), 455-460.
- Garbade, K., and W. Silber, "Price Movements and Price Discovery in Futures and Cash Markets," Review of Economics and Statistics 65 (May 1983), 289-297.
- Grauer, F., and R. Litzenberger, "The Pricing of Commodity Futures Contracts, Nominal Bonds and Other Risky Assets under Commodity Price Uncertainty," Journal of Finance 34 (March 1979), 69-83.
- Henry, A.L., D.L. Tuttle, and W.E. Young, "Market Indexes and their Implications for Portfolio Management," Financial Analysis Journal (September - October 1971), 75-85.
- Hirshleifer, J., "Speculation and Equilibrium: Information, Risk and Market," Quarterly Journal of Economics 89 (November 1975), 520-542.
- Jensen, M.C., Financial Analysis Journal 23 (November - December 1967), 77-85.
- Johnson, L.L., "The Theory of Hedging and Speculation in Commodity Futures," Review of Economic Studies 28 (June 1960), 139-151.
- Johnston, J. Econometric Methods. McGraw-Hill Book Inc., New York (1984).
- Latane, H.A., and W.E. Young, "Test of Portfolio Building Rules," Journal of Finance 24 (September 1969), 595-612.
- Lintner, J., "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," Review of Economics and Statistics 47 (February 1965), 13-37.
- Levy, H., "The CAPM and Beta in an Imperfect Market," Journal of Portfolio Management 6 (Winter 1980), 5-11.
- Mayers, D., "Nonmarketable Assets and Capital Market Equilibrium under Uncertainty," in Studies in the Theory of Capital Markets, edited by Michael C. Jensen. Praeger, New York (1972).
- Modest, D.M., and M. Sundaresan, "The Relationship Between Spot and Futures Prices in Stock Index Futures Markets: Some Preliminary Evidence," Journal of Futures Markets 3 (Spring 1983), 15-41.
- Mossin, J., "Equilibrium in a Capital Asset Market," Econometrica 34 (October 1966), 768-783.

- Peck, A., "Hedging and Income Stability: Concepts, Implications and Example," American Journal of Agricultural Economics 57 (August 1975).
- Raynauld, J., and T. Jacques, "Risk Premiums in Futures Markets: An Empirical Investigation," Journal of Futures Markets 4 1984, 189-211.
- Pinduck, R.S., and D.L. Rubinfeld, Econometric Models and Econometric Forecase. McGraw - Hill Book Co., New York (1981).
- Pratt, J.W., "Risk Aversion in the Small and in the Large," Econometrica 32 (January-April 1964), 122-136.
- Radcliffe, R.C., Investment: Concepts, Analysis and Strategy. Scott, Foresman and Company, Glenview, Illinois (1982)
- Richard, S.F., and M. Sundaresan, "A Continuous Time Equilibrium Model of Forward Price and Futures Price in a Multigood, Economy," Journal of Financial Economics 9 (1980), 347-372.
- Ross, S.A., "Options and Efficiency," Quarterly Journal of Economics (February 1976), 75-89.
- Samuelson, P.A., "Proof that Properly Anticipated Price Fluctuations Randomly," Industrial Management Review (Spring 1965), 41-49.
- Samuelson, P.A., "Is Real-World Price a Tale Told by the Idiot of Chance?" Review of Economic and Statistics 58 (February 1976), 120-123.
- Sarnat, M., "Capital of Optimal Portfolios," Journal of Finance 29 (September 1974), 1241-1253.
- Sarris, A., "A Bayesian Approach to Estimation of Time-Varying Regression Coefficients," Annals of Economic and Social Measurement 2 (October 1973), 501-523.
- Schlarbaum, G.G., W. Lewellen, and R.C. Lease, "Realized Returns on Common Stock Investments: The experience of Individual Investors," Journal of Business 51 (April 1978), 299-325.
- Sharpe, W.F., "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance 19 (September 1964), 419-442.
- Simkowitz, M.A., and W.L. Beedles, "Diversification in a Three-Movement World," Journal of Financial and Quantitative Analysis 13 (December 1978), 924-941.
- Smith, K.V., "Stock Price and Economic Indexes for Generating Efficient Portfolios," Journal of Business 41 (April 1968), 326-336.
- Stein, J.L., "The Simultaneous Determination of Spot and Futures Prices," American Economic Review 51 (December 1961), 1012-1025.

- Stoll, H., "Commodity Futures and Spot Price Determination and Hedging in Capital Market Equilibrium," Journal of Financial and Quantitative Analysis 14 (1979) 873-893.
- Telser, Lester G., "Safety First and Hedging," Review of Economic Studies 23, (1955), 1-16.
- Wu, S., "The Pricing of Index Futures," Term Paper (To Dr. A.A. Heggested), Department of Finance, University of Florida, Gainesville (May 1983).

BIOGRAPHICAL SKETCH

Soushan Wu was born on June 3, 1950, in Kaoshiung, Taiwan, Republic of China. He received his Bachelor of Commerce (in accounting and statistics) degree in 1972 from National Chung-Hsin University, Taipei, Taiwan. In 1974, he received his Master of Science degree in management science from National Chiao-Tung University, Hsinch, Taiwan. Since then, he continues to be on the faculty of accounting and information systems with National Chiao-Tung University. He also served as a consultant to Bristol-Myers, Taiwan Subsidiary, Inc. in 1979-1980.

In 1981, he entered the University of Florida to pursue his graduate study in finance. During his graduate work, he received financial support from the National Science Foundation of Republic of China, as well as student assistantship from the Department of Finance, University of Florida.

At the completion of his study, he will return to his country to continue his career in teaching and research.

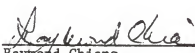
I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.


Robert C. Radcliffe, Chairman
Associate Professor of Finance,
Insurance, and Real Estate.

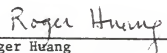
I certifiy that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.


S. R. Cosslett
Associate Professor of Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.


Raymond Chiang
Associate Professor of Finance,
Insurance, and Real Estate

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a disseration for the degree of Doctor of Philosophy.


Roger Huang
Assistant Professor of Finance,
Insurance, and Real Estate

This dissertation was submitted to the Graduate Faculty of the School of Accounting in the College of Business Administration and to the Graduate School, and was accepted as a partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December 1984

Dean for the Graduate School and
Research